

**Algebra III**

**Content Standards**

**2016**

Course Title: Algebra III

Course/Unit Credit: 1

Course Number: 439070

Teacher Licensure: Please refer to the Course Code Management System (<https://adedata.arkansas.gov/ccms/>) for the most current licensure codes.

Grades: 9-12

Prerequisite: Algebra I, Geometry, Algebra II

Algebra III

This course will enhance the higher level thinking skills developed in Algebra II through a more in-depth study of those concepts and exploration of some pre-calculus concepts. Students in Algebra III will be challenged to increase understanding of algebraic, graphical, and numerical methods to analyze, translate and solve polynomial, rational, exponential, and logarithmic functions. Modeling real world situations is an important part of this course. Sequences and series will be used to represent and analyze real world problems and mathematical situations. Algebra III will also include a study of matrices and conics. Algebra III does not require Arkansas Department of Education approval.

Prerequisites: Algebra I, Geometry, Algebra II

Strand Content Standard

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| Matrix Operations |  |
|  | 1. Students will perform operations with matrices and use them to solve systems of equations. |
| Conic Sections |  |
|  | 2. Students will identify, analyze, and sketch the graphs of the conic sections and relate the equations and graphs. |
| Function Operations and Properties |  |
|  | 3. Students will be able to find the inverse of functions and use composition of functions to prove that two functions are inverses. |
| Interpreting Functions |  |
|  | 4. Students will be able to interpret different types of functions and key characteristics including polynomial, exponential, logarithmic, and rational functions. |
| Sequences and Series |  |
|  | 5. Students will use sequences and series to represent and analyze mathematical situations. |

Strand: Matrix Operations

Content Standard 1: Students will perform operations with matrices and use them to solve systems of equations.

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| MO.1.AIII.1 | Use matrices to represent and manipulate data  (e.g., to represent payoffs or incidence relationships in a network) |
| MO.1.AIII.2 | Multiply matrices by scalars to produce new matrices  (e.g., as when all of the payoffs in a game are doubled) |
| MO.1.AIII.3 | Add, subtract, and multiply matrices of appropriate dimensions |
| MO.1.AIII.4 | Understand that, unlike multiplication of numbers, *matrix multiplication* for square matrices is not a commutative operation, but still satisfies the associative and distributive properties |
| MO.1.AIII.5 | Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers; the *determinant* of a square matrix is nonzero if and only if the matrix has a multiplicative inverse |
| MO.1.AIII.6 | Multiply a *vector* (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another *vector*; work with matrices as *transformations* of *vectors* |
| MO.1.AIII.7 | Work with matrices as *transformations* of the plane; interpret the absolute value of the *determinant* in terms of area |
| MO.1. AIII.8 | Represent a system of linear equations as a single matrix equation in a *vector variable* |
| MO.1. AIII.9 | Find the inverse of a matrix if it exists; use the inverse to solve systems of linear equations using technology for matrices of dimension or greater |

Strand: Conic Sections

Content Standard 2: Students will identify, analyze, and sketch the graphs of the conic sections and relate the equations and graphs.

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| CS.2. AIII.1 | Find the *conjugate* of a *complex number*; use *conjugate*s to find *moduli* and quotients of *complex numbers* |
| CS.2. AIII.2 | Derive the equations of ellipses and hyperbolas given the *foci*, using the fact that the sum or difference of distances from the *foci* is constant; find the equations for the *asymptotes* of a hyperbola |
| CS.2. AIII.3 | Complete the square in order to generate an equivalent form of an equation for a *conic section;* use that equivalent form to identify key characteristics of the *conic section* |
| CS.2. AIII.4 | Identify, graph, write, and analyze equations of each type of *conic section*, using properties such as *symmetry*, intercepts, *foci*, *asymptotes*, and *eccentricity*, and using technology when appropriate |
| CS.2. AIII.5 | Solve systems of equations and inequalities involving conics and other types of equations, with and without appropriate technology |

Strand: Function Operations and Properties

Content Standard 3: Students will be able to find the inverse of functions and usecomposition of functions to prove that two functions are inverses.

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| FOP.3.AIII.1 | Compose functions  (e.g., if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time) |
| FOP.3. AIII.2 | Verify, by *composition,* that one function is the inverse of another |
| FOP.3. AIII.3 | Read values of an *inverse function* from a graph or a table, given that the function has an inverse |
| FOP.3. AIII.4 | Produce an *invertible function* from a *non-invertible function* by restricting the domain |
| FOP.3. AIII.5 | Combine standard function types using arithmetic operations  (e.g., build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential and relate these functions to the model) |
| FOP.3. AIII.6 | Understand the inverse relationship between exponents and *logarithms;* use this relationship to solve problems involving *logarithms* and exponents |
| FOP.3.AIII.7 | Graph *transformations* of functions including quadratic, absolute value, square root, cube root, cubic, and step functions; graph *piece-wise* defined functions including these *transformations* |
| FOP.3. AIII.8 | Determine numerically, graphically, and algebraically if a function is even, odd, or neither |

Strand: Interpreting Functions

Content Standard 4: Students will be able to interpret different types of functions and key characteristics including polynomial, exponential, logarithmic, and rational functions.

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| IF.4. AIII.1 | Graph rational functions identifying zeros and *asymptotes* when suitable factorizations are available; show end behavior |
| IF.4. AIII.2 | Analyze and interpret polynomial functions numerically, graphically, and algebraically, identifying key characteristics such as intercepts, end behavior, domain and range, relative and absolute *maximum* and *minimum*, as well as intervals over which the function increases and decreases |
| IF.4. AIII.3 | Analyze and interpret rational functions numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes* (*vertical, horizontal*, and slant), end behavior, point discontinuities, intercepts, and domain and range |
| IF.4. AIII.4 | Analyze and interpret *exponential functions* numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes*, end behavior, intercepts, and domain and range |
| IF.4. AIII.5 | Analyze and interpret *logarithmic functions* numerically, graphically, and algebraically, identifying key characteristics such as *asymptotes*, end behavior, intercepts, and domain and range |
| IF.4. AIII.6 | Build functions to model real-world applications using algebraic operations on functions and *composition of functions,* with and without appropriate technology  [e.g., profit functions as well as volume and surface area (*optimization* subject to constraints)] |

Strand: Sequences and Series

Content Standard 5: Students will use sequencesand series to represent and analyze mathematical situations.

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| SS.5. AIII.1 | Write *arithmetic* and *geometric sequences* both recursively and with an explicit formula; translate between the two forms |
| SS.5. AIII.2 | Use *arithmetic* and *geometric sequences* both recursively and with an explicit formula to model situations |
| SS.5. AIII.3 | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers |
| SS.5. AIII.~~4~~ | Use sequences and series to solve real world problems, with and without appropriate technology |

Glossary for Algebra III

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| Arithmetic Sequence | A sequence in which each term after the first is found by adding a constant, called the common difference to the previous term |
| Asymptote(s) | Line(s) to which a graph becomes arbitrarily close as the value of or increases or decreases without bound  (vertical, horizontal, slant) |
| Complex number(s) | Number(s) that can be written as the sum or difference of a real number and an imaginary number (e.g., ) |
| Composition of Functions | Suppose *f* and *g* are functions such that the range of *g* is a subset of the domain of *f*, then the composite function *f* of *g* can be described by the equation ◦ |
| Conic section | Any figure that can be formed by slicing a double cone with a plane |
| Conjugate(s) | The result of writing the sum of two terms as a difference or vice-versa |
| Determinant | A single number obtained from a matrix that reveals a variety of the matrix's properties |
| Eccentricity | A number that indicates how drawn out or attenuated a conic section is; eccentricity is represented by the letter (no relation to ...) |
| Exponential Function | A function in which the variable(s) occurs in the exponent [e.g., x ] |
| Foci | Two fixed points on the interior of an ellipse used in the formal definition of the curve |
| Geometric Sequence | A sequence in which each term after the first is found by multiplying the previous term by a constant, called the common ratio, |
| Horizontal Asymptote | A horizontal line to which a graph becomes arbitrarily close as the value of increases or decreases without bound |
| Inverse Function | Two functions and are inverse functions, if and only if, both of their *composition*s yield the identity function; for example,◦ ◦ |
| Invertible function | A function that has an inverse |
| Logarithmic Function | A function of the form b, where and |
| Logarithm(s) | The logarithm base b of a number is the power to which must be raised in order to equal ; this is written b (e.g., 3 4 ) |

Glossary for Algebra III

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| Matrix multiplication | Given Matrix *A* with dimensions and Matrix *B* with dimensions ; the matrix multiplication of *AB* results in a matrix with dimensions of and is only possible if the number of columns in *A* is equal to the number of rows in *B* (if and only if ); matrices are multiplied as shown below:    Example: |
| Maximum | The greatest value of a function if the function has such an extreme value |
| Minimum | The least value of a function if the function has such an extreme value |
| Moduli | Distance of the complex number from the origin in a complex plane |
| Non-invertible function | A function that does not have an inverse |
| Optimization | The process by which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within a domain |
| Piece-wise Function | A function defined by different rules for different parts of the domain |
| Symmetry | A figure has symmetry if the figure and its image coincide after a transformation |
| Transformations of graphs | Operations that alter the form of a figure  (e.g., horizontal shifts, vertical shifts, horizontal stretches, vertical stretches, reflections) |
| Transformations of vectors | For each point in the plane there is a corresponding position vector from the origin to the point The motion of the point under a geometric transformation (for example, a rotation or reflection) can be modeled as the multiplication of a 2 by 2 transformation matrix times the position vector |
| Vector | A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers |
| Vector (variable) | A vector whose components are variable. For example, if a system of linear equations is written in matrix form , where is the coefficient matrix and is the vector of the constants, is an unknown vector variable |
| Vertical Asymptote | A vertical line to which a graph becomes arbitrarily close as the value of ) increases or decreases without bound |