ACTAAP

Arkansas Comprehensive Testing, Assessment, and Accountability Program

Teacher Handbook

Geometry
End-of-Course Examination

April 2007 Administration

Arkansas Department of Education

Teacher Handbook—2007 Geometry

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Introduction—2007 Geometry

The **Arkansas Comprehensive Testing, Assessment, and Accountability Program (ACTAAP)** includes an April End-of-Course Examination for students completing Geometry or the equivalent by the end of the spring semester. The examination consists of multiple-choice and open-response questions that directly assess student knowledge. The Arkansas *Geometry Mathematics Curriculum Framework* is the basis for the development of the *Geometry End-of-Course Examination.*

In April 2007, all students who had completed or were completing the required course work for Geometry by the end of the spring semester participated in the *Geometry End-of-Course Examination.* Results of the *Geometry End-of-Course Examination* will be provided to all students, schools, and districts to be used as the basis for instructional change.

The handbook provides information about the scoring of student responses to the Geometry open-response items. It describes the scoring procedures and the scoring criteria (rubrics) used to assess student responses. Copies of actual student responses are provided, along with scores given to those responses, to illustrate how the scoring criteria were applied to Geometry open-response items.

Additional information about the *Geometry End-of-Course Examination* is available through the Arkansas Department of Education. Questions can be addressed to Dr. Gayle Potter at 501-682-4558.

Scoring Student Responses to Open-Response Items—2007 Geometry

The multiple-choice and open-response test items for the *Geometry End-of-Course Examination* are developed with the assistance and approval of the Geometry Content Advisory Committee. This committee is composed of active Arkansas educators with expertise in Mathematics education. The Geometry Content Advisory Committee develops and reviews multiple-choice and open-response items to ensure that they reflect the Arkansas *Geometry Mathematics Curriculum Framework.*

While multiple-choice items are scored by machine to determine if the student chose the correct answer from four options, open-response items must be scored by trained "readers" using a pre-established set of scoring criteria.

Reader Training

Before readers are allowed to begin assigning scores to any student responses, they go through intensive training. The first step in that training is for the readers to read the Geometry open-response items as they appear in the test booklet and to respond—just as the student test takers are required to do. This step gives the readers some insight into how the students might have responded. The next step is the readers' introduction to the scoring rubric. All of the specific requirements of the rubric are explained by the Scoring Director who has been specifically trained to lead the scoring group. Then responses (anchor papers) that illustrate the score points of the rubric are presented to the readers and discussed. The goal of this discussion is for the readers to understand why a particular response (or type of response) receives a particular score. After discussion of the rubric and anchor papers, readers practice scoring sets of responses that have been pre-scored and selected for use as training papers. Detailed discussion of the responses and the scores they receive follows.

After three or four of these practice sets, readers are given "qualifying rounds." These are additional sets of pre-scored papers, and, in order to qualify, each reader must score in exact agreement on at least 80% of the responses and have no more than 5% non-adjacent agreement on the responses. Readers who do not score within the required rate of agreement are not allowed to score the *Geometry End-of-Course Examination* responses.

Once scoring of the actual student responses begins, readers are monitored constantly throughout the project to ensure that they are scoring according to the criteria. Daily and cumulative statistics are posted and analyzed, and Scoring Directors or Team Leaders reread selected responses scored by the readers. These procedures promote reliable and consistent scoring. Any reader who does not maintain an acceptable level of agreement is dismissed from the project.

Scoring Procedures

All student responses to the *Geometry End-of-Course Examination* open-response test items are scored independently by two readers. Those two scores are compared, and responses that receive scores that are non-adjacent (a "1" and a "3," for example) are scored a third time by a Team Leader or the Scoring Director for resolution.

On the following pages, open-response items are presented as they appeared in the April 2007 *Geometry End-of-Course Examination*. The specific scoring rubric for each item and annotated responses for each score point of the rubric follows. The goal is for classroom teachers and their students to understand how responses are scored. It is hoped that this understanding will help students see what kind of performance is expected of them on the *Geometry End-of-Course Examination*.

GEOMETRY
RESPONSES

Item A—2007 Geometry

The coordinates for the vertices of quadrilateral GHIJ are given in the table below.

Vertex	Coordinates
G	(−4, 0)
H	(0, 3)
I	(4, 1)
J	(4, −4)

1. On the grid provided in your answer document, plot the vertices of quadrilateral GHIJ. Connect the vertices to form a quadrilateral.

2. Find the slopes of $\overline{GJ}$ and $\overline{HI}$ and use the slopes to make a conclusion about the relationship between the segments. Show all of your work and/or explain your answer.

3. Find the lengths of $\overline{GH}$ and $\overline{IJ}$ and use the lengths of the segments to make a conclusion about the relationship between the segments. Show all of your work and/or explain your answer.

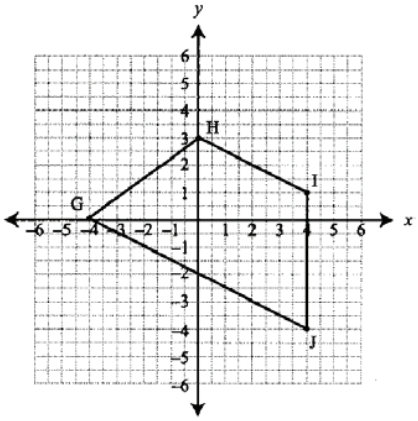
4. What type of quadrilateral is GHIJ? Be as specific as possible and use your answers from Parts 2 and 3 to support your answer.

Item A Scoring Rubric—2007 Geometry

SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work.
3	The student earns 3½–4½ points.
2	The student earns 2–3 points.
1	The student earns ½–1½ points, or some minimal understanding is shown. Ex: One correct slope in Part 2 with no work shown.
0	The student earns 0 points. No understanding is shown. Ex: $IJ = 5$, if points I and J are plotted correctly with work missing.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

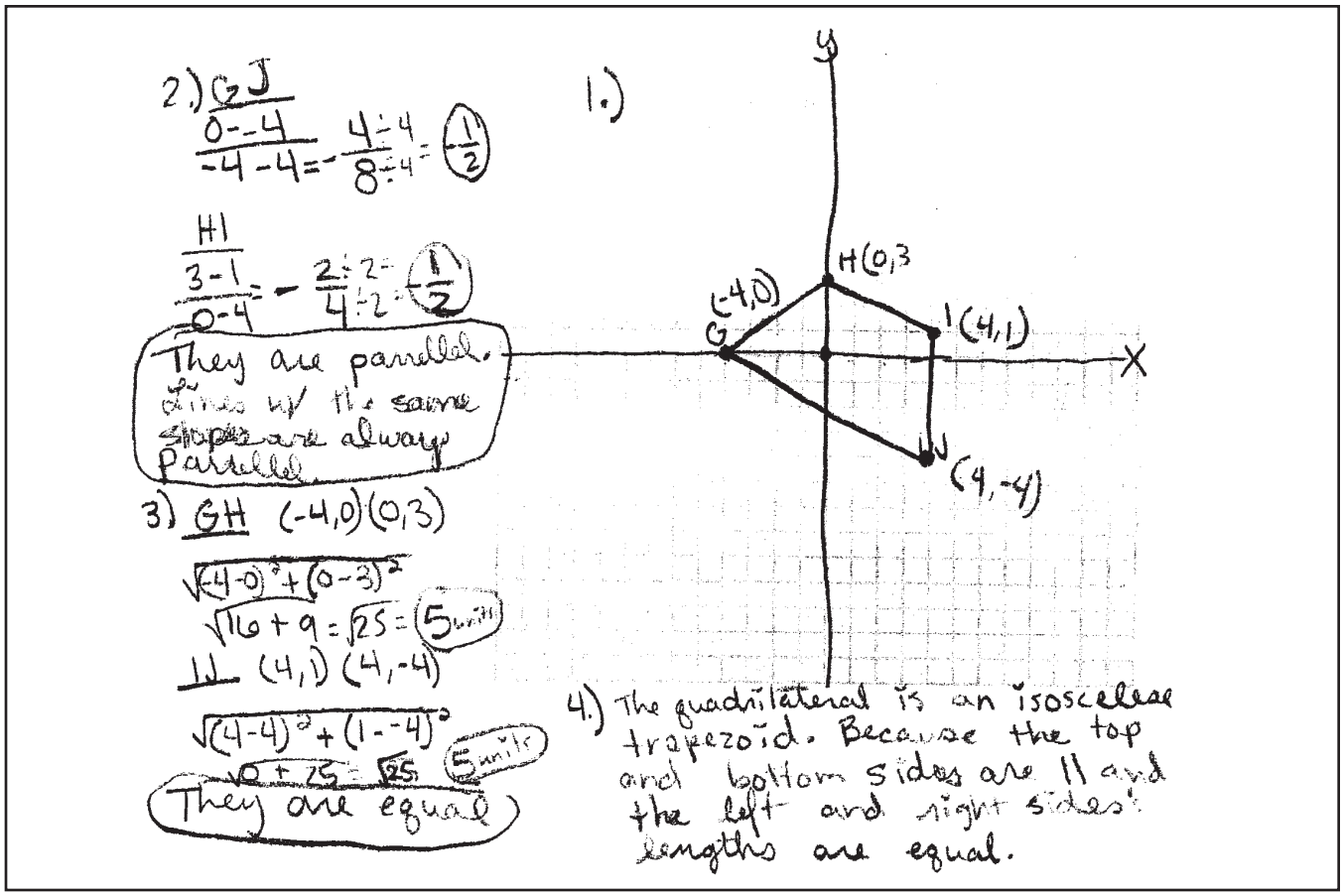
Item A Solution and Scoring—2007 Geometry

Solution and Scoring

Part	Points
1	**1 point possible** 1 point: **Correctly plotted and labeled quadrilateral *GHIJ*, as shown below.** Note: Labels may be missing but not incorrect.  OR ½ point: **Graph contains one error but is otherwise correct.** Ex: Label(s) are incorrect; points are correctly plotted. Ex: One point is off due to a counting or copy error. Ex: Intervals are inconsistent.
2	**1 point possible** 1 point: **Response includes the following:** • Both slopes are $-\frac{1}{2}$ (or equivalent). • Correct work is shown for at least one slope. Ex: Slope of $\overline{GJ} = \frac{0--4}{-4-4} = \frac{4}{-8} = -\frac{1}{2}$ Ex: Slope of $\overline{HI} = \frac{3-1}{0-4} = \frac{2}{-4} = -\frac{1}{2}$ Ex: Student uses graph to show the slope is down 1 unit and right 2 units for each segment. • Relationship: "The segments are parallel (since they have the same slope)," or "the segments have the same slope." OR ½ point: **Response is incomplete and/or contains a calculation or copy error.** Ex: Slopes are $-\frac{1}{2}$, work is shown, but the relationship statement is missing or incorrect. Ex: Slope(s) are incorrect due to a calculation or copy error. Correct procedures are used and work is shown. Relationship may be missing. Ex: Two correct slopes; work is missing. Ex: One correct slope with work; other slope is missing or incorrect. **Note: No credit is given for relationship statement if slopes are incorrect or missing.**

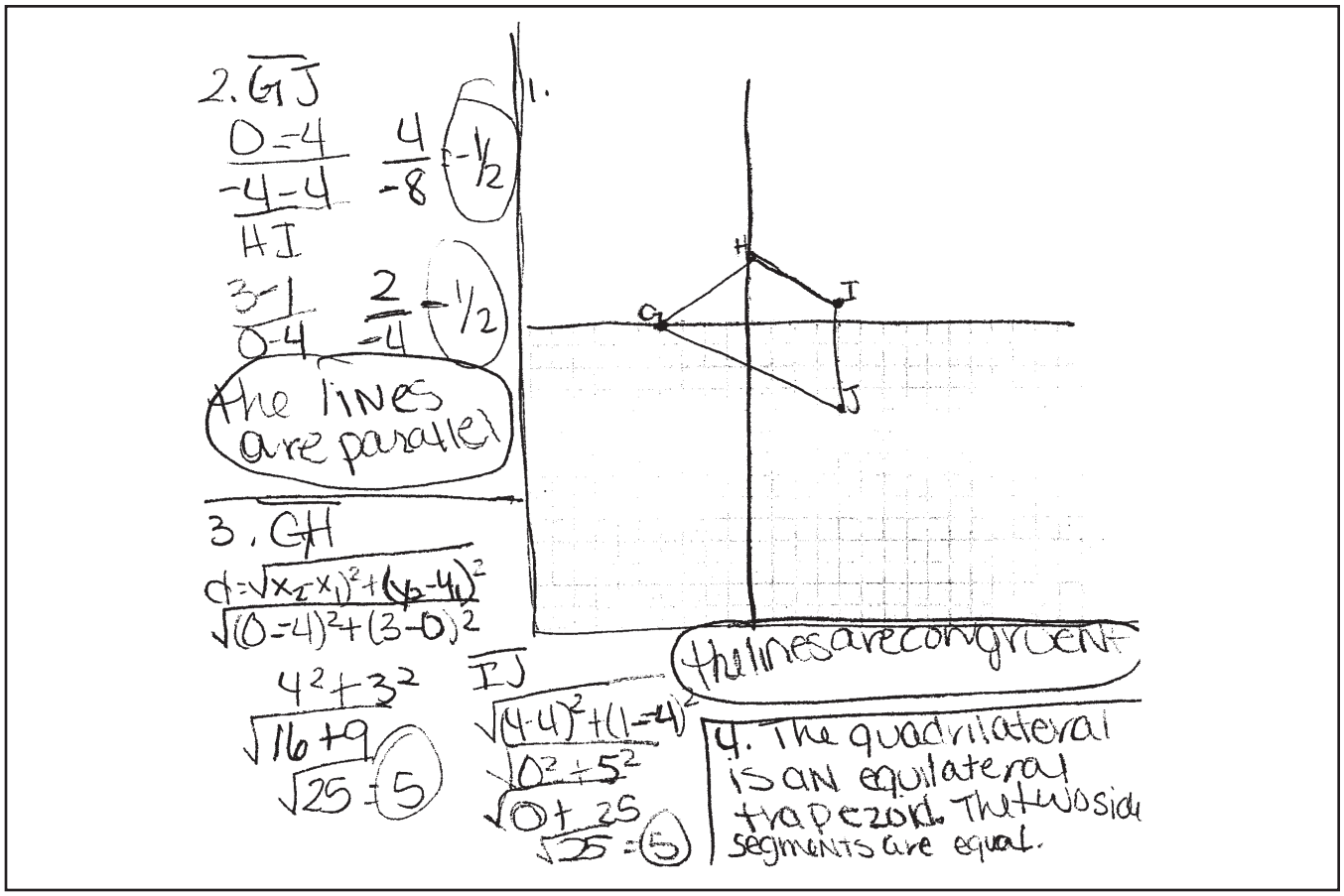
Solution and Scoring (continued)

Part	Points
3	**2 point possible** 2 points: **Response includes the following:** • Both lengths are 5 units. • Work is correct and complete for at least *GH*: Ex: GH $= \sqrt{(-4-0)^2+(0-3)^2} = \sqrt{4^2+3^2} = \sqrt{25} = 5$ IJ $= \sqrt{(4-4)^2+(1--4)^2} = \sqrt{0^2+5^2} = \sqrt{25} = 5$ Ex: Student uses graph to find *GH* by creating a right triangle with *GH* as the hypotenuse and using the Pythagorean Theorem to find its length. *IJ* is found by counting the units since it is a vertical segment. • Relationship: "The sides are congruent (or they have the same length)." OR 1 point: **Response is incomplete and/or contains a calculation or copy error.** Give credit for the following or equivalent: • Lengths are 5, work is shown, but relationship statement is missing or incorrect, or • Length(s) are incorrect due to a calculation or copy error. Correct procedures are used and work is shown. Relationship may be missing, or • Both lengths are 5, work is missing; with or without relationship, or • One length is 5 with work shown, second length (and/or procedure) is missing or incorrect; with or without relationship. **Note: No credit is given for relationship statement if lengths are incorrect or missing.**
4	**1 point possible** **Note: No credit is given for "Trapezoid" without correct or equal slopes in Part 2. No credit is given for "Isosceles Trapezoid" without correct or equal answers (slopes & lengths) in Parts 2 and 3.** **Note: Do not give credit for "Trapezoid" if reasoning (if given) is not based on slopes or is incorrect. Do not give credit for "Isosceles Trapezoid" if reasoning (if given) is not based on slopes and lengths or is incorrect.** 1 point: **Correct (specific) name with correct and complete support.** • "Isosceles trapezoid because it has one pair of parallel sides and the other pair of sides is congruent." OR ½ point: **Give credit for any of the following:** • Correct (specific) name with missing, incomplete, or vague (but not incorrect) support. Ex: Isosceles trapezoid—one pair of sides are parallel. Ex: Isosceles trapezoid. • Correct (not specific) name. Support, if given, must be correct. Ex: Trapezoid—one pair of sides is parallel and the other pair is $\cong$. Ex: Trapezoid.



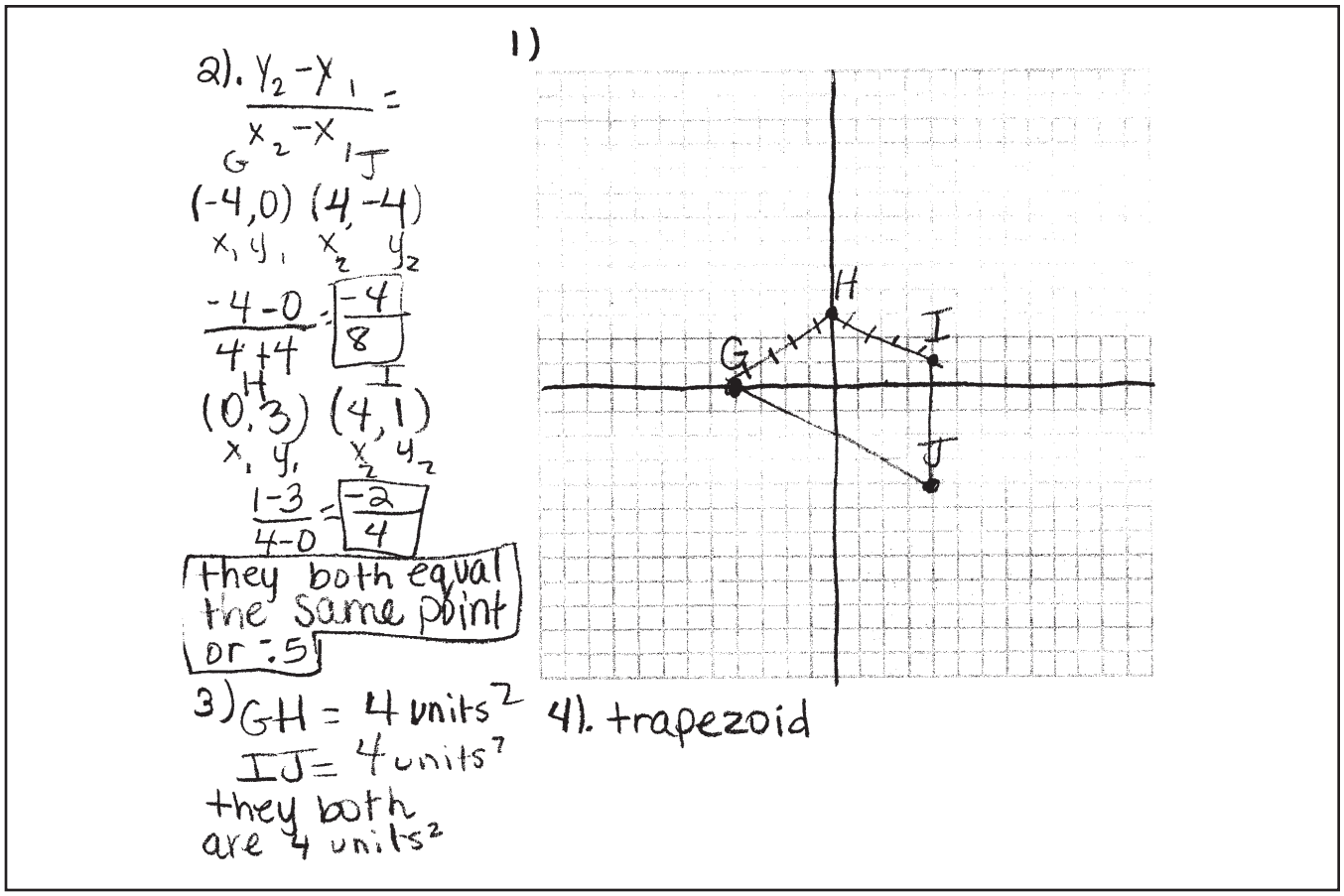
SCORE: 4

Part 1:		**Points**
Correct graph:	Points correctly plotted and connected	1
Part 2:		
Correct answers:	Both slopes = $-\frac{1}{2}$	
Correct procedure:	$\frac{0-(-4)}{-4-4} = \#$ $\quad \frac{3-1}{0-4} = \#$	
Correct relationship:	"They are parallel."	1
Part 3:		
Correct answers:	$\overline{GH} = \overline{IJ} = 5$	
Correct procedure:	$\sqrt{(-4-0)^2 + (0-3)^2} = \#$	
	$\sqrt{(4-4)^2 + (1-(-4))^2} = \#$	
Correct relationship:	"They are equal"	2
Part 4:		
Correct answer:	Isosceles trapezoid	
Correct explanation:	"...top and bottom sides are ‖ and the left and ride sides' lengths are equal."	1
TOTAL POINTS:		**5**



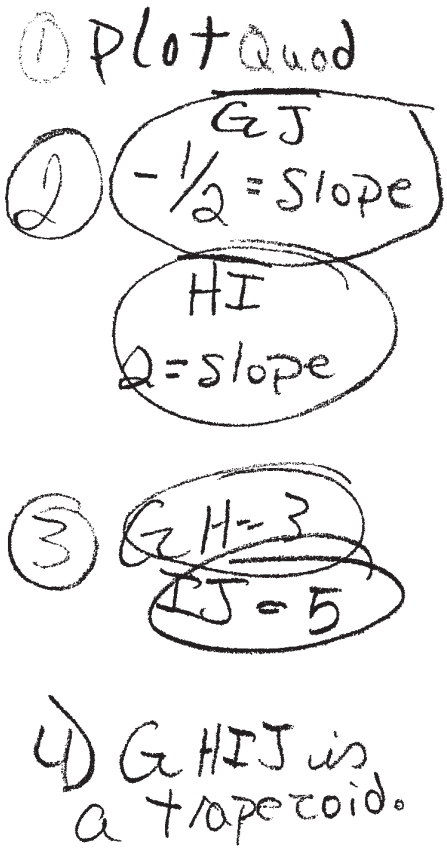
SCORE: 3

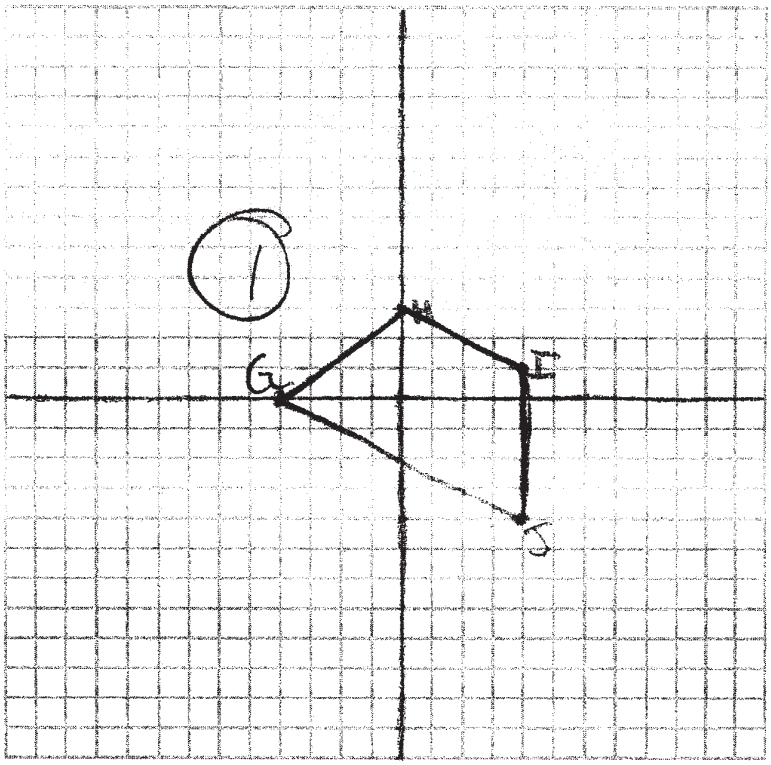
Part 1:		**Points**
Correct graph:	Points correctly plotted and connected	1
Part 2:		
Correct answers:	Both slopes = $-\frac{1}{2}$	
Correct procedure:	$\frac{0-\text{-}4}{\text{-}4-4}=\frac{4}{\text{-}8}=\#$ $\quad\frac{3-1}{0-4}=\frac{2}{\text{-}4}=\#$	
Correct relationship:	"the lines are parallel"	1
Part 3:		
Correct answers:	$\overline{GH}=\overline{IJ}=5$	
Correct procedure:	$\sqrt{(0-\text{-}4)^2+(3-0)^2}=\#$	
	$\sqrt{(4-4)^2+(1-\text{-}4)^2}=\#$	
Correct relationship:	"the lines are congruent"	2
Part 4:		
Incorrect answer:	Equilateral trapezoid	-
TOTAL POINTS:		**4**



SCORE: 2

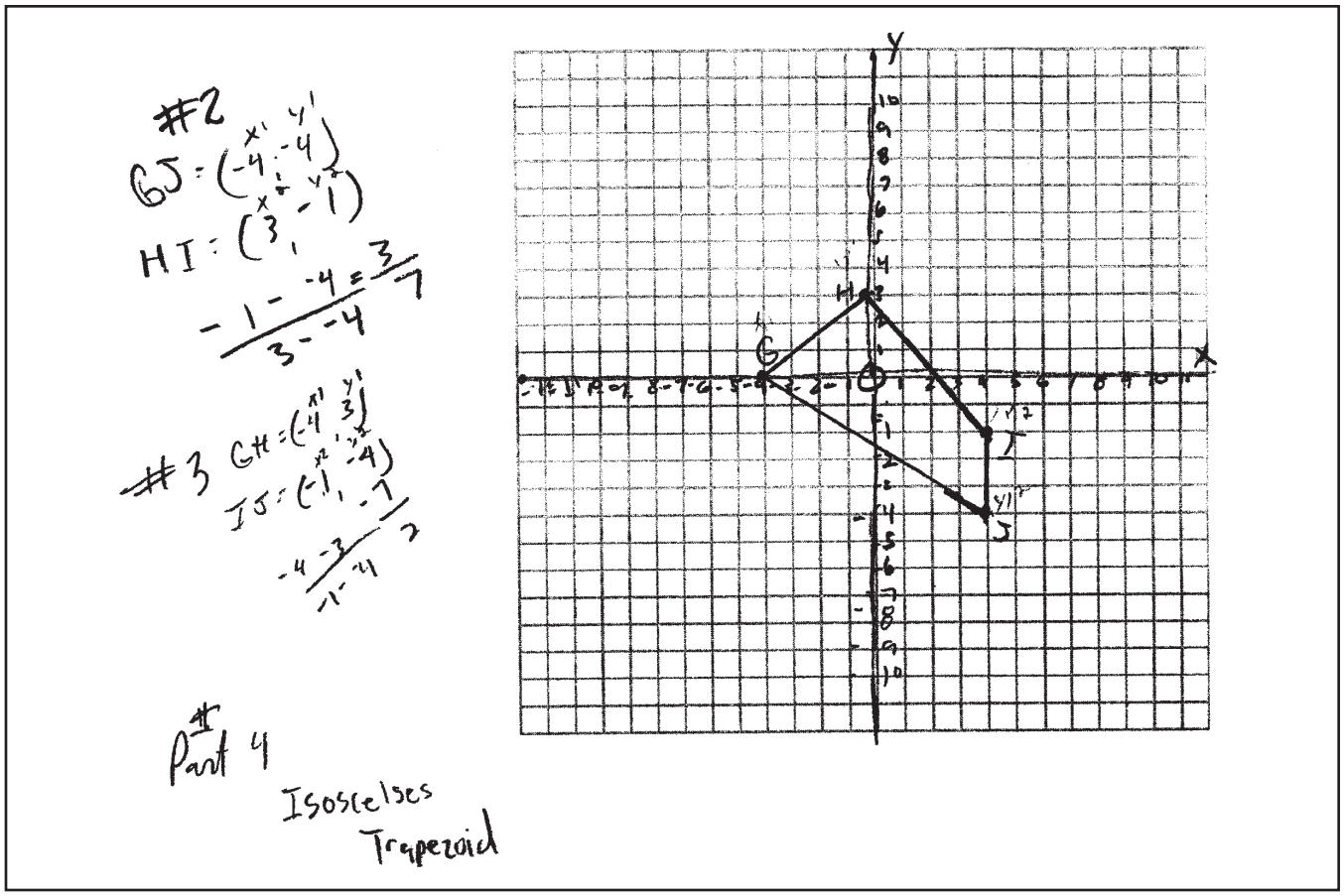
Part 1:		**Points**
Correct graph:	Points correctly plotted and connected	1
Part 2:		
Correct answers:	Both slopes = $-\frac{1}{2}$	
Correct procedure:	$\frac{-4-0}{4+4} = \frac{-4}{8}$ $\quad$ $\frac{1-3}{4-0} = \frac{-2}{4}$	
Incorrect relationship:	"they both equal the same point"	½
Part 3:		
Incorrect answers:	$GH = IJ = 4$ units^2	
Missing procedure:		-
Part 4:		
Correct answer:	Trapezoid	
Missing explanation:		½
TOTAL POINTS:		**2**





SCORE: 1

Part 1:		**Points**
Correct graph:	Points correctly plotted and connected	1
Part 2:		
1 correct answer: Missing procedure:	Slope $\overline{GJ} = -\frac{1}{2}$	
1 incorrect answer: Missing procedure:	Slope $\overline{HI} = 2$	-
Part 3:		
1 correct answer: Missing procedure:	$IJ = 5$	
1 incorrect answer: Missing procedure:	$GH = 3$	-
Part 4:		
Answer irrelevant since slopes were not equal in Part 2.		-
TOTAL POINTS:		**1**

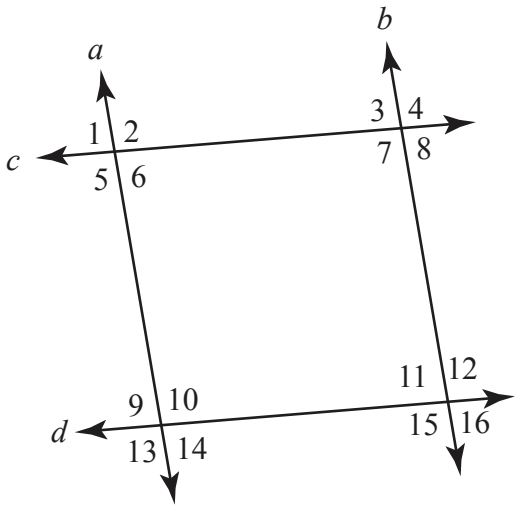


SCORE: 0

Part 1:		**Points**
Incorrect graph:	Point *I* incorrectly plotted Intervals inconsistent on *y*-axis	-
Part 2:		
Incorrect answer:	$\frac{3}{7}$	
Incorrect procedure:	$\frac{-1 - -4}{3 - -4} = \#$	-
Part 3:		
Incorrect answer:	$\frac{-7}{2}$	
Incorrect procedure:	$\frac{-4 - 3}{-1 - -4} = \#$	-
Part 4:		
Answer irrelevant since Part 2 answer was incorrect.		-
TOTAL POINTS:		**0**

Item B—2007 Geometry

Lines a, b, c, and d are shown in a plane below.



1. Name one angle that must be congruent to $\angle 1$ in order to prove that $a \parallel b$. Explain your reasoning.

2. Name one angle that must be congruent to $\angle 1$ in order to prove that $c \parallel d$. Explain your reasoning.

3. If $a \parallel b$ and $c \parallel d$, what is the relationship between $\angle 9$ and $\angle 4$? Explain your reasoning.

4. The measure of $\angle 13$ equals $(6x + 40)$ degrees and the measure of $\angle 16$ equals $(120 - 4x)$ degrees. If $a \parallel b$, what is the value of x? Show your work and/or explain your answer.

Item B Scoring Rubric—2007 Geometry

SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work.
3	The student earns 3½–4½ points.
2	The student earns 2–3 points.
1	The student earns ½–1 ½ points, or some minimal understanding is shown. Ex: Correct angle is given in Parts 1 and 2 with incorrect explanation. Ex: Correct relationship is given in Part 3 with incorrect explanation.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

Item B Solution and Scoring—2007 Geometry

Solution and Scoring

Part	Points
1	**1 point possible** 1 point: **Correct answer ($\angle 3$ or $\angle 8$) with correct explanation.** Give credit for the following or equivalent: • "$\angle 3$; If two lines are cut by a transversal and the corresponding angles are congruent, then the lines are parallel," or • "$\angle 8$; If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel." Note: The converse of either of the above theorems is acceptable, or • "Angle 3**.** They are corresponding angles," or • "$\angle 8$; Because it's the opposite exterior angle," or • " Angle 8 because it's vertical to angle 3 and 1 and 3 are $\cong$." OR ½ point: **One correct answer with vague or missing, but not incorrect, explanation.** Give credit for the following or equivalent: Ex: "Angle 3. It's in the same place…" Ex: "Angle 8 because it's a vertical angle." Ex:"$\angle 3$."
2	**1 point possible** 1 point: **One correct answer ($\angle 9$ or $\angle 14$) with correct explanation.** Give credit for the following or equivalent: • "$\angle$ 9; If two lines are cut by a transversal and the corresponding angles are congruent, then the lines are parallel," or • "$\angle 14$; If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel." Note: The converse of either of the above theorems is acceptable, or • "Angle 9**.** They are corresponding angles," or • "$\angle 14$; Because it's the opposite exterior angle," or • "Angle 14 because it's vertical to angle 9 and 1 and 9 are $\cong$." OR ½ point: **Correct answer with vague or missing, but not incorrect, explanation.** Give credit for the following or equivalent: Ex: "Angle 9 since if you shifted it over they're the same…" Ex: "Angle 14 because it's vertical." Ex: "$\angle$ 9."

Solution and Scoring (continued)

Part	Points
3	**1 point possible** 1 point: **Correct relationship with correct and complete explanation.** Note: You must be able to follow the logic step by step. Names of angle relationships are not required. Give credit for the following or equivalent: Ex: Angle 9 and Angle 4 are supplementary: Angle 9 ≅ Angle 1 ≅ Angle 3 Angle 3 and Angle 4 are supplementary Therefore, Angle 9 and Angle 4 are supplementary. Ex: "Angles 9 & 4 are supplementary because 3 and 4 are supplementary" (Angle 3 ≅ Angle 1 ≅ Angle 9 given in Parts 1 & 2) Ex: "9 and 4 are supplementary: $\angle 10 \cong \angle 2 \cong \angle 4$, 9 & 10 are supplementary and 3 & 4 are supplementary, so 9 & 4 are too." OR ½ point: **Correct answer with incomplete, vague, or missing, but not incorrect, explanation.** Give credit for the following or equivalent: Ex: "9 and 4 are supplementary since 10 ≅ 4." Ex: "9 and 4 are supplementary because they add up to 180 degrees." Ex: "9 & 4 add up to 180 º."
4	**2 points possible** 1 point: **Correct value for x: 10.** Note: Do not give credit if procedure is incorrect AND 1 point: **Correct and complete procedure shown and/or explained.** Work may contain a calculation or copy error Give credit for the following or equivalent: • $(6x + 40) + (120 – 4x) = 180$ (initial equation is adequate) $2x + 160 = 180$ $2x = 20$ $x = \#$.

1. In order for a||b ∠1 must be congruent to ∠8 because they are alternate exterior angles and if the lines are parallel alternate exterior angles are congruent
2. In order for c||d ∠1 must be congruent to ∠14 because they are alternater exterior angles and if the lines are parallel, alternate exterior angles are congruent
3. ∠9 and ∠4 are supplementary. ∠9 and ∠16 are alternate exterior angles so they are congruent. ∠4 and ∠16 are consecutive exterior angles so they are supplementary. Since ∠9 and ∠16 are congruent ∠9 must also be supplementary to ∠4.
4. ∠13 and ∠16 are consecutive exterior angles so they are supplementary. If you add them together they equal 180°. $6x+40+120-4x=180$ $2x+160=180$ $2x=20$ $x=10$

SCORE: 4

Part 1:		**Points**
Correct angle with correct explanation:	∠8 "alternate exterior angles"	1
Part 2:		
Correct angle with correct explanation:	∠14 "alternater exterior angles"	1
Part 3:		
Correct relationship with correct/complete explanation:	∠9 and ∠4 are supplementary ∠9 ≅ ∠16 (alternate exterior) ∠4 & ∠16 (consecutive exterior) → ∠9 & ∠4 are supplementary	1
Part 4:		
Correct answer:	$x = 10$	1
Correct/complete procedure:	$6x + 40 + 120 – 4x = 180$, etc.	1
TOTAL POINTS:		**5**

① ∠3 must be congruent to ∠1 in order for a∥b. The reason is that if the two angles are different, then the two lines will be at different angles and will intersect.

② ∠9 must be congruent to ∠1 for c∥d. The reason is b/c if the angles are different then the lines will tilt towards each other and will eventually intersect.

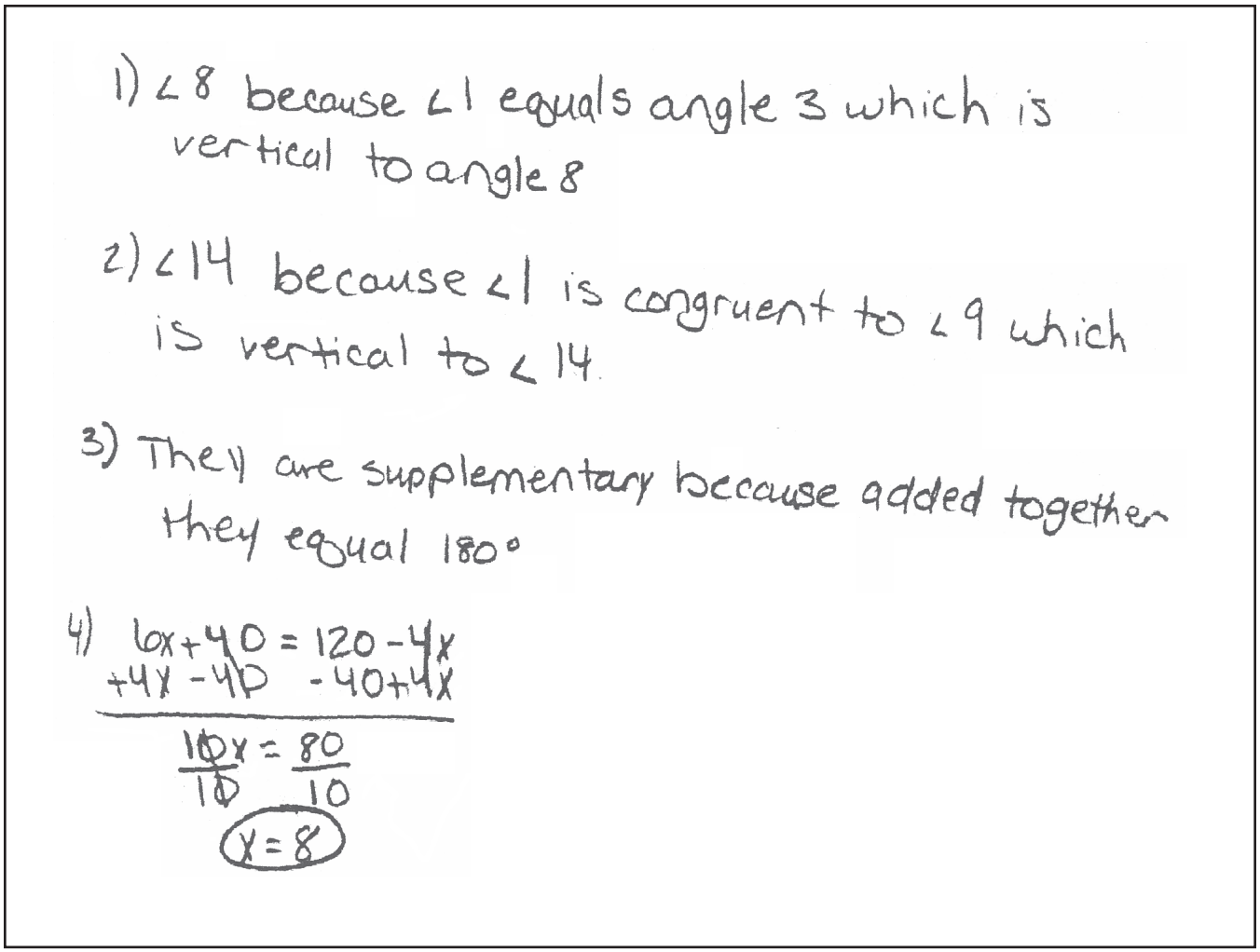
③ ∠9 and ∠4 must be supplementary, which means they add up to 180°. The reason is that ∠3 and ∠4 are supplementary and ∠3 ≅ ∠9.

④ $x = 10$

$180 = \angle 13 + \angle 16$
$180 = (6x+40) + (120-4x)$
$180 = 2x + 160$
$20 = 2x$
$10 = x$

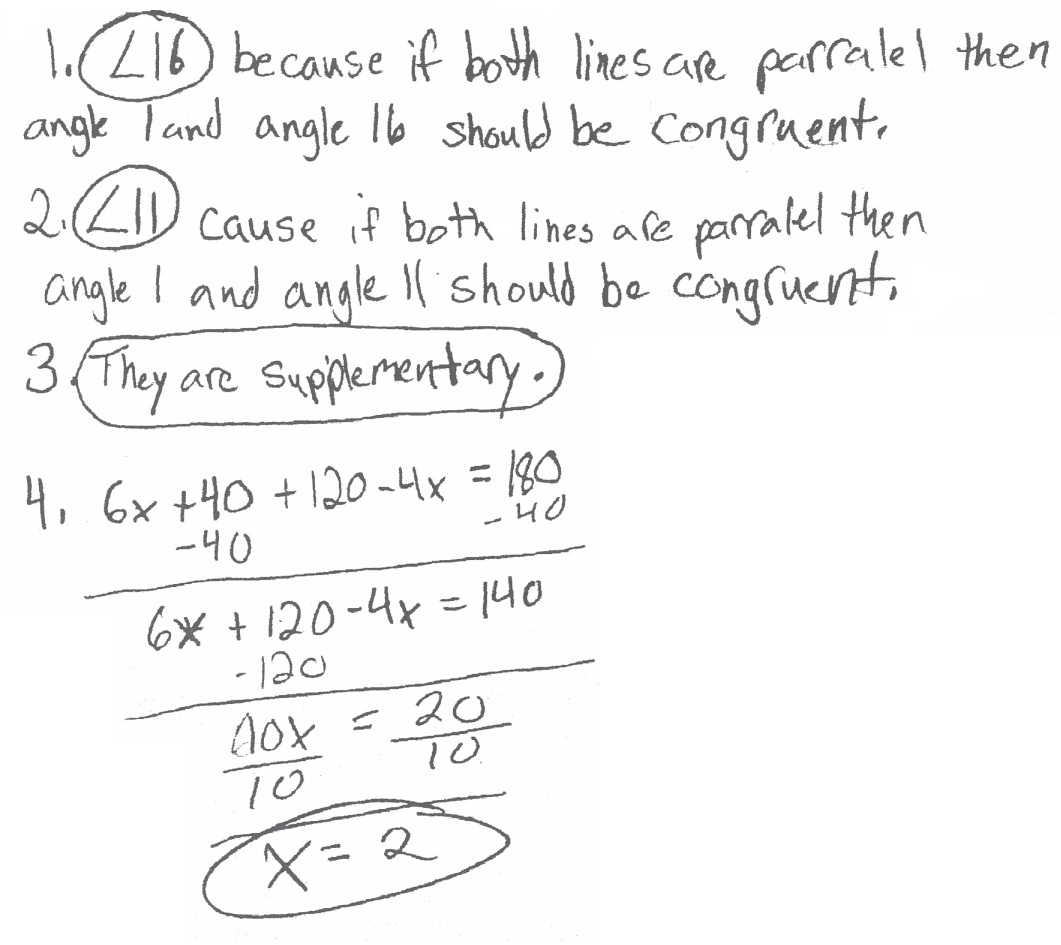
SCORE: 3

Part 1:		**Points**
Correct angle with vague explanation:	∠3 "if the two angles are different"	½
Part 2:		
Correct angle with vague explanation:	∠9 "if the angles are different"	½
Part 3:		
Correct relationship with correct/complete explanation:	∠9 and ∠4 are supplementary ∠3 & ∠4 are supplementary ∠3 ≅ ∠9 (Parts 1 & 2)	1
Part 4:		
Correct answer:	$x = 10$	1
Correct/complete procedure:	$6x + 40 + 120 – 4x = 180$, etc.	1
TOTAL POINTS:		**4**



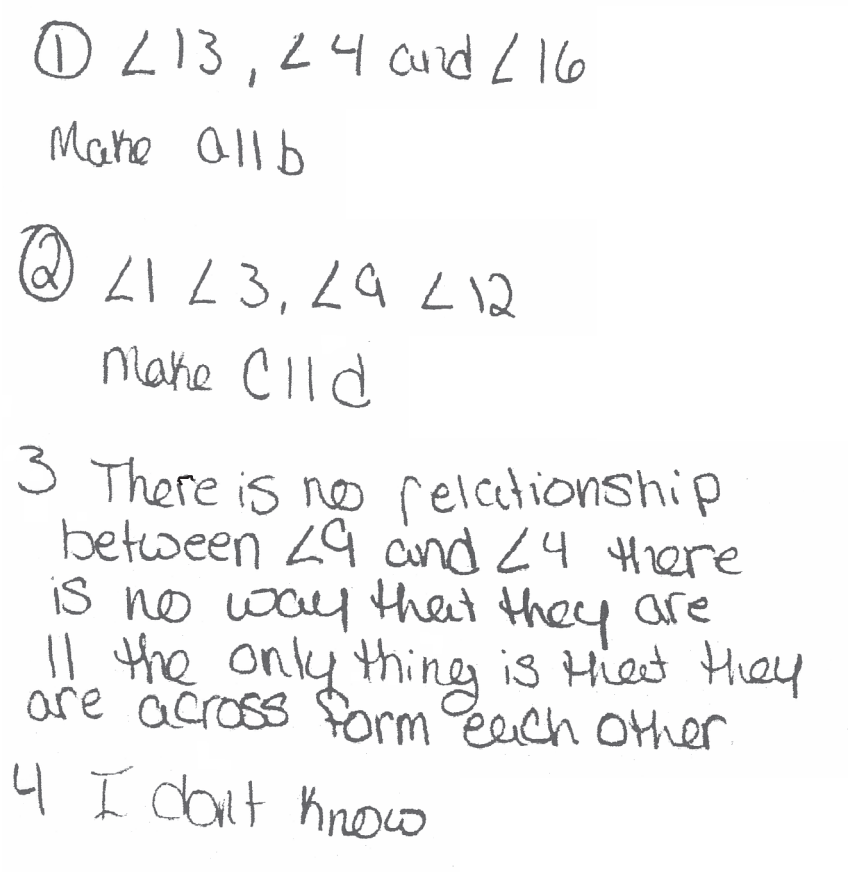
SCORE: 2

Part 1:		**Points**
Correct angle with correct explanation:	∠8 "∠1 equals angle 3 which is vertical to angle 8"	1
Part 2:		
Correct angle with correct explanation:	∠14 "∠1 is congruent to ∠9 which is vertical to ∠14"	1
Part 3:		
Correct relationship with missing explanation:	"They are supplementary because added together they equal 180°"	½
Part 4:		
Incorrect answer:	$x = 8$	-
Incorrect procedure:	$6x + 40 = 120 – 4x$	-
TOTAL POINTS:		**2½**



SCORE: 1

Part 1:		**Points**
Incorrect angle:	∠16	-
Part 2:		
Incorrect angle:	∠11	-
Part 3:		
Correct relationship with missing explanation:	"They are supplementary."	½
Part 4:		
Incorrect answer due to calculation error:	$x = 2$	
Correct/complete procedure:	$6x + 40 + 120 - 4x = 180$, etc. ($6x - 4x = 10x$ calculation error)	- 1
TOTAL POINTS:		**1½**

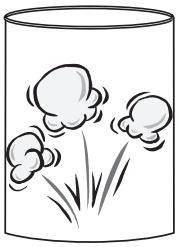


SCORE: 0

Part 1:		**Points**
Incorrect angle:	∠13, ∠4, ∠16	-
Part 2:		
1 correct angle with	∠9	
2 incorrect angles:	∠3, ∠12	-
Part 3:		
Incorrect relationship:	"no relationship"	-
Part 4:		
Missing answer:		-
Missing procedure:		-
TOTAL POINTS:		**0**

Item C—2007 Geometry

A movie theater sells popcorn in 3 different sizes of cylindrical tubs.

	 Small	 Medium	 Large
Cost	$2.50	$4.50	$8.25
Radius	3 inches		
Height	6 inches		
Volume			
Number of Servings	2 servings		

1. In your answer document, copy and complete the table above using the following information. Show all of your work and/or explain your answer.

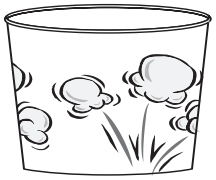
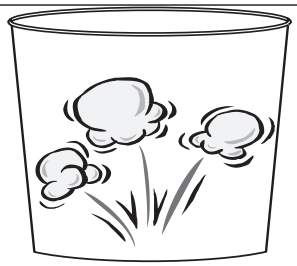
   - The medium tub has the same radius as the small tub, but it holds 1.5 times as much popcorn as the small tub.
   - The radius of the large tub of popcorn is 1.5 times the radius of the small tub. The height of the large tub is 1.5 times the height of the small tub.

2. A family of 6 people goes to the movies. What is the **least** expensive way to purchase a serving of popcorn for each person? Show all of your work and/or explain your answer.

Item C Scoring Rubric—2007 Geometry

SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work. Units are not required, but must be correct if included in student work.
3	The student earns 3½–4½ points.
2	The student earns 2–3 points.
1	The student earns ½–1½ points, or some minimal understanding is shown.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

Solution and Scoring

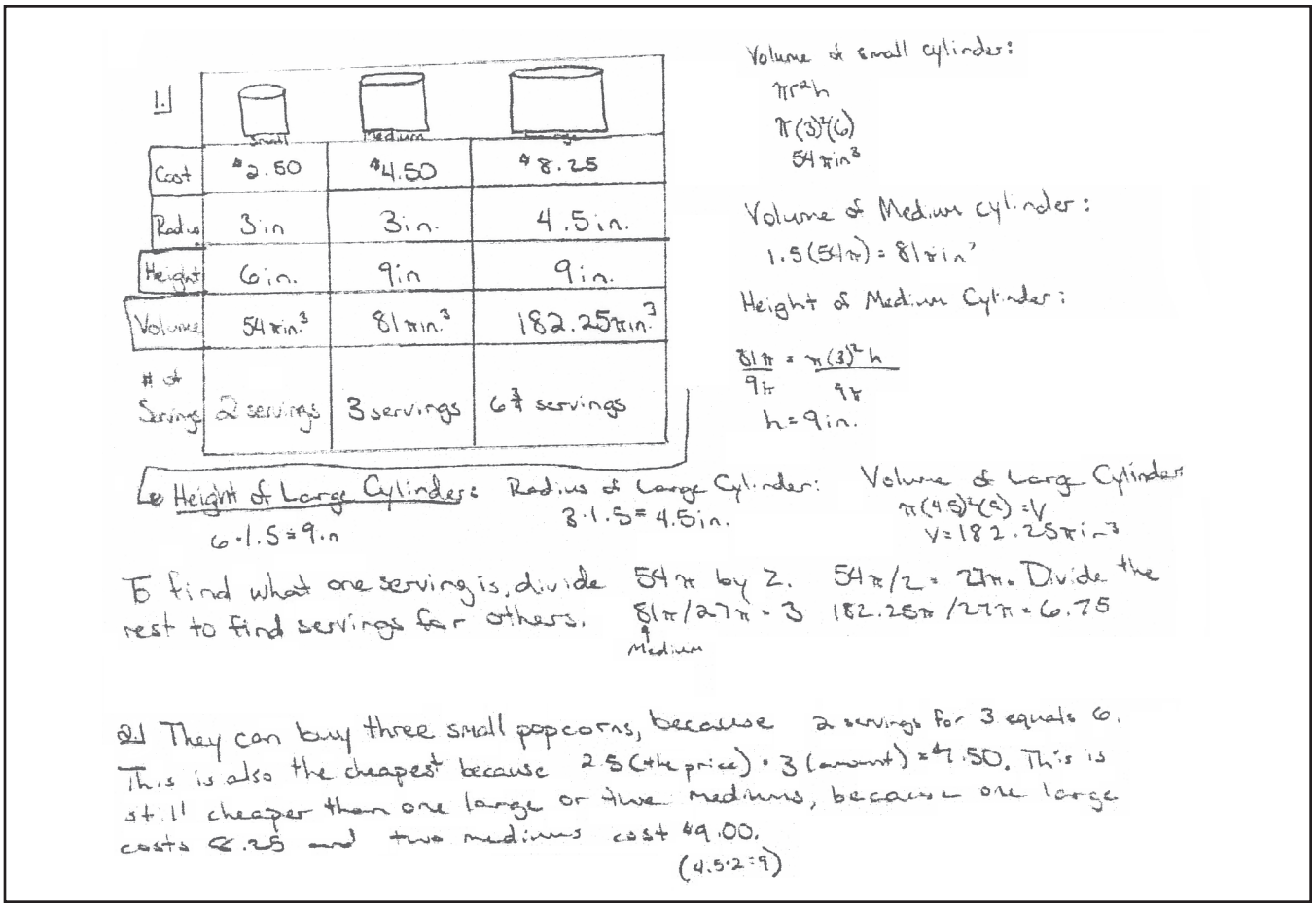
	 Small	 Medium	 Large
Cost	**$2.50**	**$4.50**	**$8.25**
Radius	3 in. (Given)	**3 (in.)**	**4.5 (in.)** (1.5 x 3 may be missing)
Height	6 in. (Given)	**9 (in.)** $\frac{255}{(3^2)\pi}$ (work may be missing depending upon method)	**9 (in.)** (1.5 x 6 may be missing)
Volume	**54π or 169 – 170(in.³)** **$V = \pi(3^2)6$**	**81π or 253.5 – 255 (in.³)** **V = 170(1.5) or $V = \pi(3^2)9$**	**182.25π or 572 – 573 (in.³)** **$V = \pi(4.5^2)(9)$**
Number of Servings	2 servings (Given)	**3 servings** $\frac{170}{2} = 85$, $\frac{255}{85} = 3$ or equivalent (may be missing)	**6, 6.75, or 7 servings (may be rounded or truncated)** $\frac{170}{2} = 85$, $\frac{573}{85} = 6.75$ or equivalent

Solution and Scoring (continued)

Part	Points
1	**4 points possible** **Radii, Heights, Volumes** 3 points: **Correct radii, heights, and volumes for all three sizes.** Correct procedure shown for finding at least one volume. Work for finding radii and heights is not required. OR 2 points: **Give credit for the following:** • Correct radii, heights, and volumes for all three sizes with no work shown, or • Answer(s) are incorrect due to one calculation error with correct procedure used and work shown. OR 1 ½ points: **Give credit for the following:** • Correct radii, heights, and volumes for two of the three sizes with correct procedure used and work shown for at least one volume. OR 1 point: **Give credit for the following:** • Correct radii, heights, and volumes for two of the three sizes with no work shown, or • Correct procedures shown (volume) for two out of three sizes using correct radii and heights. **# of Servings** Note: Number of servings must be based on volumes that are given. 1 point: **Correct number of servings (based on volumes) for Medium and Large sizes.** • Correct procedure used and work is shown for Large size, or • Proportional reasoning is shown for at least Medium size. OR ½ point: **Give credit for the following:** • Correct # of servings (based on volumes) for Medium and Large with no work shown, or • Number of serving(s) incorrect due to a calculation error with correct procedure used and work shown. Note: To check number of servings, divide volume by 85 or V(small) ÷ 2. Answer may be rounded.

Solution and Scoring (continued)

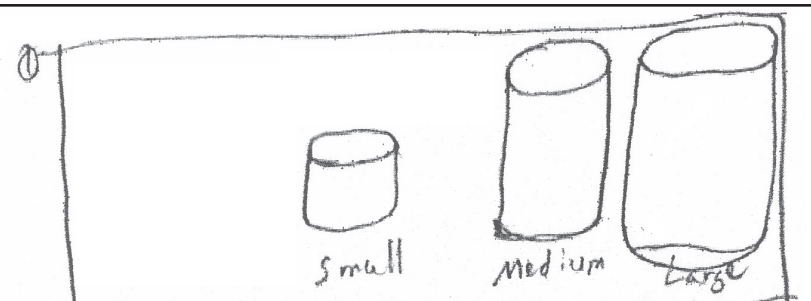
Part	Points
2	**1 point possible** 1 point: **Give credit for the following or equivalent:** • Least expensive choice (based on number of servings given for the three sizes) with correct and complete procedure (three options for comparison). Ex: Three Small: \$7.50, 2 Medium: \$9.00, 1 Large: \$8.25, so they should buy 3 small tubs. Ex: Cost per serving: Small: \$2.50/2 servings = \$1.25 per serving Medium: \$4.50/3 servings = \$1.50 per serving Large: \$8.25/6 = \$1.375 per serving Three small tubs would be the least expensive. Ex: Cost per serving: Small: \$2.50/2 servings = \$1.25 per serving Medium: \$4.50/3 servings = \$1.50 per serving Large: \$8.25/6.75 servings = \$1.22 per serving A large container is the least expensive. OR ½ point: **Response is incomplete or contains one calculation error.** Give credit for the following or equivalent: • Least expensive choice (based on number of servings given for the three sizes) with work shown for two options for comparison, or • Correct and complete procedure (three options)—least expensive choice is missing or incorrect. Note: Do not give credit for least expensive choice with only one option listed.



SCORE: 4

Part 1:				**Points**
Correct radii, heights, volumes for all sizes—correct work for volume.				3
	S	**M**	**L**	
Radius:	3 in.	**3 in.**	**4.5** (1.5 × 3) in.	
Height:	6 in.	**9 in.**	**9** (1.5 × 6) in.	
Work:		**81π ÷ π9**		
Volume:	**54π in.³**	**81π in.³**	**182.25π in.³**	
Work:	**π(3²)(6)**	**(1.5)(54π)**	**π(4.5²)(9)**	
Correct # of Servings:	2 servings	**3 servings**	**6.75 servings**	
Work:	**$\frac{54\pi}{2} = 27\pi$**	**$\frac{81\pi}{27\pi}$**	**$\frac{182.25\pi}{27\pi}$**	
	(per serving)			1
Part 2:				
Correct answer with 3 correct options:	3 small popcorns 3 small: 7.50, 2 medium: 9.00, 1 large: 8.25			1
TOTAL POINTS:				**5**

No incorrect units are included.



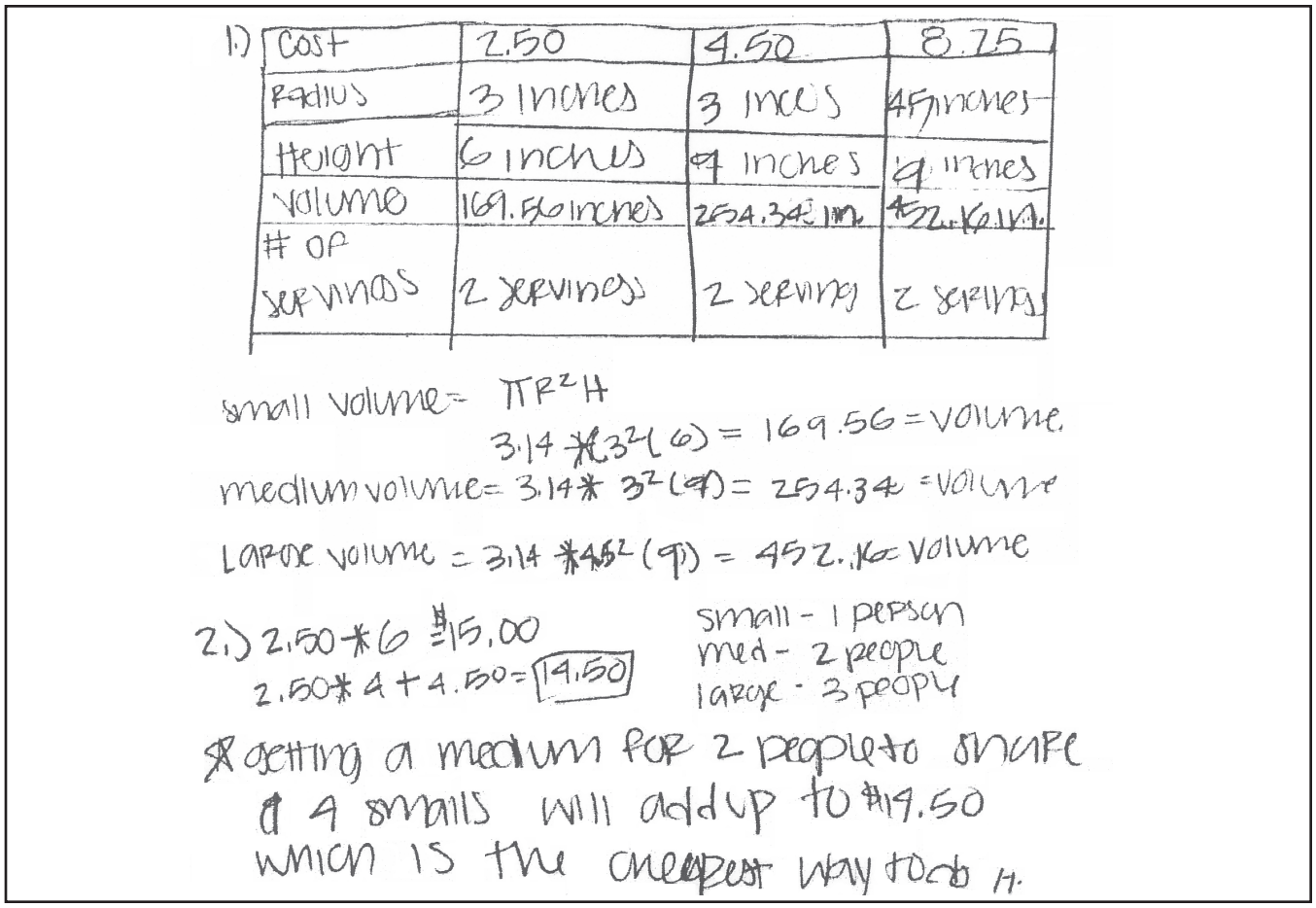
cost	$2.50	$4.50	$8.25
Radius	3 in.	3in.	4.5in.
Height	6in.	9in.	9 in.
Volume	169.56in.	254.34	572.265
Number of servings	2 servings	3 servings	6.75 servings

② 3 tubs of small = $7.50
2 tubs of medium = $9.00
1 tub of large = $8.25

3 tubs of small popcorn would be the cheapest and most efficient option.

SCORE: 3

Part 1:				**Points**
Correct radii, heights, volumes for all sizes—work is missing.				2
	S	**M**	**L**	
Radius:	3 in.	**3 in.**	**4.5 in.**	
Height:	6 in.	**9 in.**	**9 in.**	
Volume:	**169.56 in.**	**254.34**	**572.265**	
Work missing:				
Correct # of Servings:	2 servings	**3 servings**	**6.75 servings**	½
Work missing:				
Part 2:				
Correct answer with 3 correct options:	3 small popcorns 3 small: 7.50, 2 medium: 9.00, 1 large: 8.25			1
TOTAL POINTS:				**3½**



1.)

Cost	2.50	4.50	8.75
Radius	3 inches	3 inches	4.5 inches
Height	6 inches	9 inches	9 inches
Volume	169.56 inches	254.34 in.	452.16 in.
# of servings	2 servings	2 servings	2 servings

small volume = $\pi r^2 h$

$3.14 * 3^2(6) = 169.56 = volume$

medium volume = $3.14 * 3^2(9) = 254.34 = volume$

large volume = $3.14 * 4.5^2(9) = 452.16 = volume$

2.) $2.50 * 6 = \$15.00$

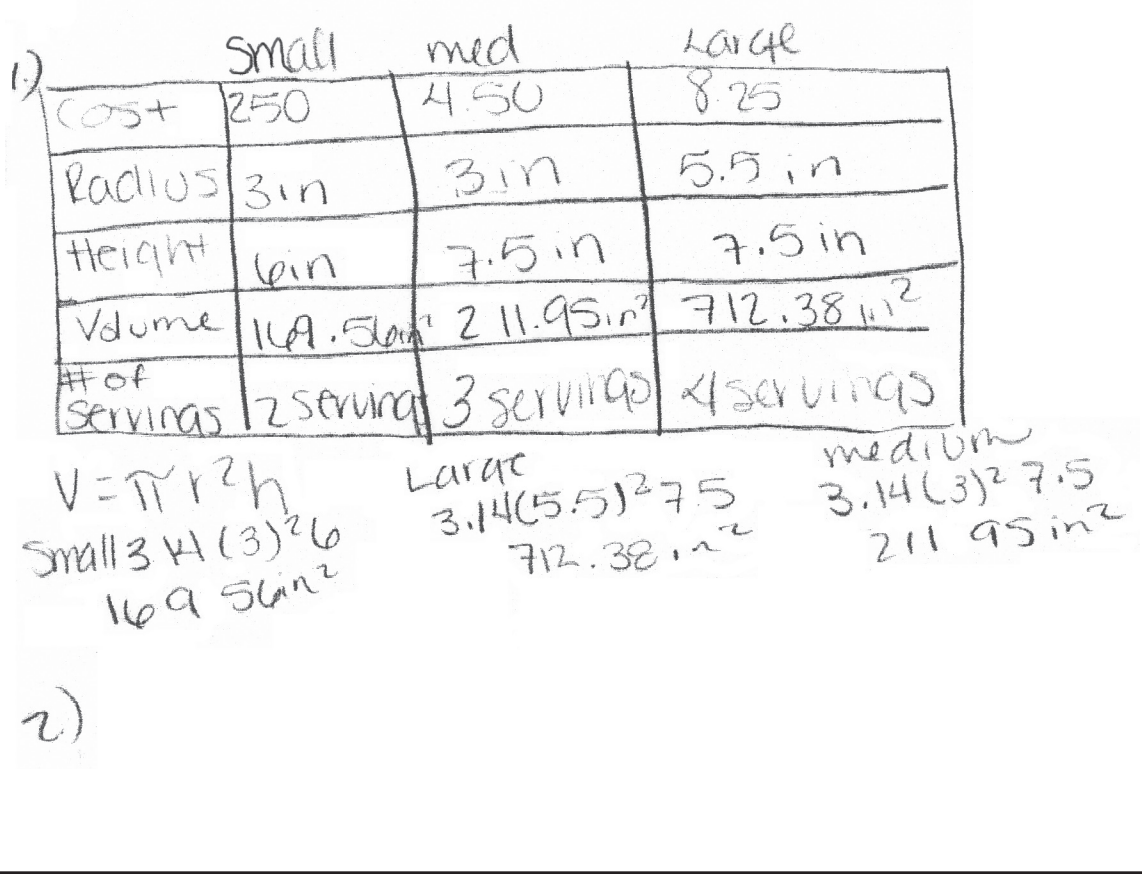
$2.50 * 4 + 4.50 = \boxed{14.50}$

small - 1 person
med - 2 people
large - 3 people

* getting a medium for 2 people to share & 4 smalls will add up to $14.50 which is the cheapest way to do it.

SCORE: 2

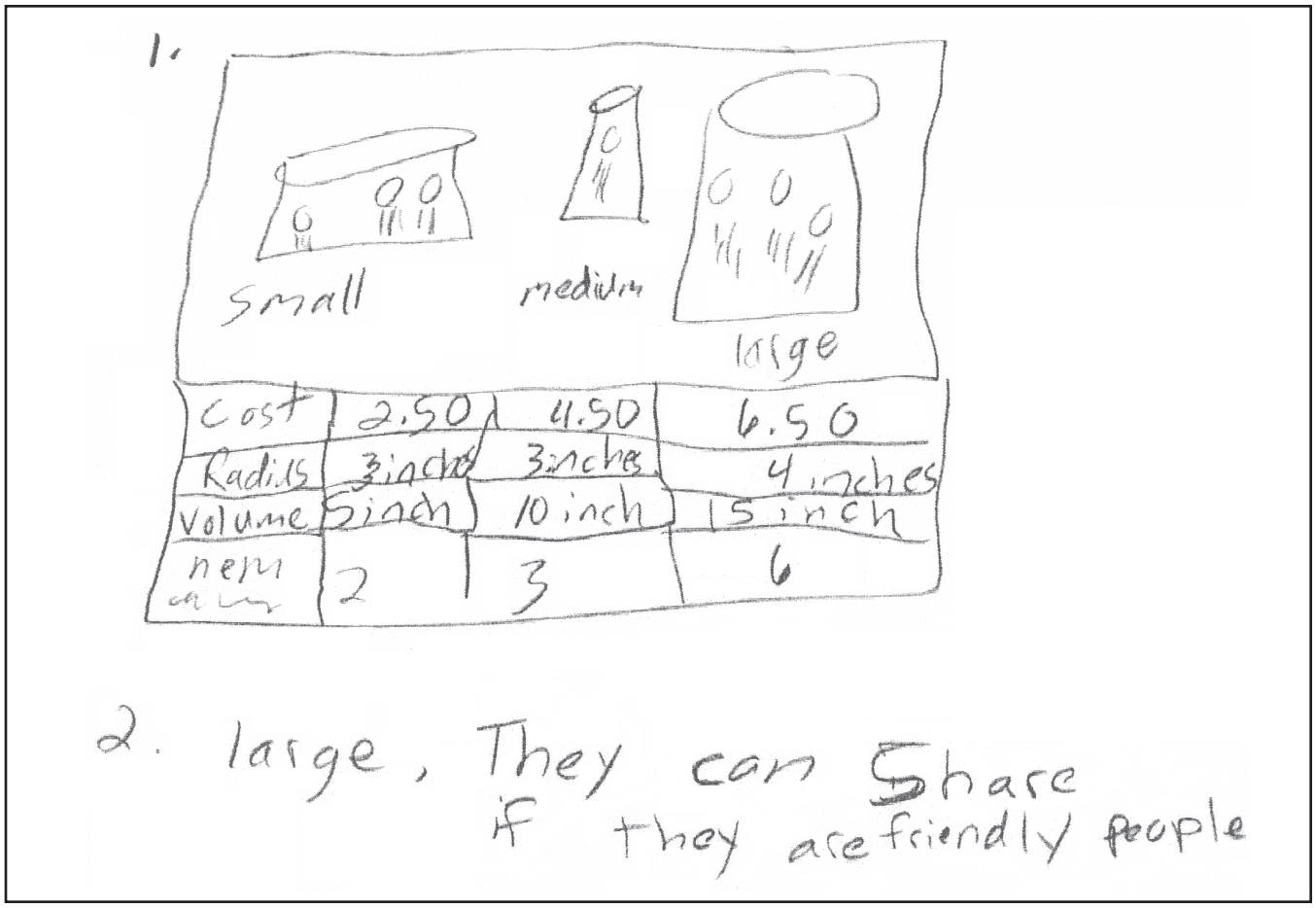
Part 1:				**Points**
Work contains one calculation error—correct procedures used.				2
	S	**M**	**L**	
Radius:	3 inches	**3 inches**	**4.5 inches**	
Height:	6 inches	**9 inches**	**9 inches**	
2/3 correct volumes:	**169.56 inches**	**254.34 inches**	**452.16 inches (calculation error)**	
Work:	**$3.14*3^2(6)$**	**$3.14*3^2(9)$**	**$3.14*4.5^2(9)$**	
Incorrect # of Servings:	2 servings	**2 servings**	**2 servings**	
Work missing:				-
Part 2:				
Incorrect answer based on # of servings:	Medium and 4 smalls			-
TOTAL POINTS:				**2**



SCORE: 1

Part 1:				**Points**
Incorrect radii, heights, volumes for all sizes—correct work for volume.				-
	S	**M**	**L**	
Radius:	3 in	**3 in**	**5.5 in (incorrect)**	
Height:	6 in	**7.5 in (incorrect)**	**7.5 in (incorrect)**	
1 correct volume:	**169.56 in²**	**211.95 in² (incorrect)**	**712.38 in² (incorrect)**	
Correct procedure:	**3.14(3²)(6)**	Correct procedure based on #s		
1 Incorrect # of Servings:	2 servings	**3 servings**	**4 servings**	
Work missing:			**(incorrect for V)**	-
Part 2:				
Answer missing:				-
TOTAL POINTS:				**0**

Although no actual points are awarded, the response receives a score of "1" since minimal understanding is shown in Part 1: 1 correct volume is found (169.56) with work shown. Also, the correct procedure is used to find volumes for M and L based on incorrect numbers.



SCORE: 0

Part 1:				**Points**
Response does not have correct #s for 2 out of 3 sizes				-
	S	**M**	**L**	
Radius:	3 inches	**3 inches**	**4 inches (incorrect)**	
Missing heights:				
Incorrect volumes:	**5 inch**	**10 inch**	**15 inch (incorrect)**	
Work missing:	**(incorrect)**	**(incorrect)**		
1 Incorrect # of Servings: (based on volume)	2	**3**	**6 (incorrect for V)**	
Work missing:				-
Part 2:				
Incorrect answer:	Large			-
TOTAL POINTS:				**0**

Item D—2007 Geometry

Jason, Patrick, and Bryce are learning to weld metal in their Industrial Technology class. Their project is to weld together metal rods of **different** lengths in order to form triangles. Their teacher gives each boy 4 rods of varying lengths, as shown in the table below.

Name	Lengths of Rods (in inches)			
Jason	5	3	12	4
Patrick	16	15	4	9
Bryce	7	2	10	19

Each boy can choose 3 of his rods to weld together, but they must form a triangle.

1. Explain the theorem that the boys can use in order to determine which 3 of their 4 rods should be welded together to make the triangle.

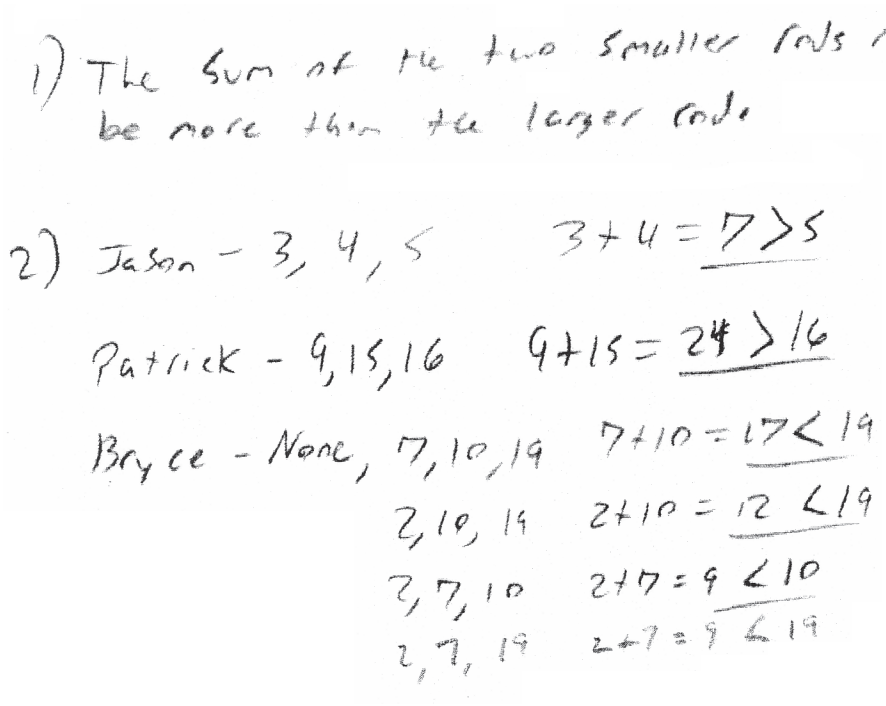
2. For each boy, determine one possible combination of his 3 metal rods that can be welded together to form the triangle. Show that this combination obeys the theorem from Part 1. If there are no combinations for any one of the boys, provide work to support your conclusion.

Item D Scoring Rubric—2007 Geometry

SCORE	DESCRIPTION
4	The student earns 4 points. The response contains no incorrect work. Units are not required, but must be correct if included in student work.
3	The student earns 3–3½ points.
2	The student earns 2–2½ points.
1	The student earns ½–1½ points, or some minimal understanding is shown.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

Solution and Scoring

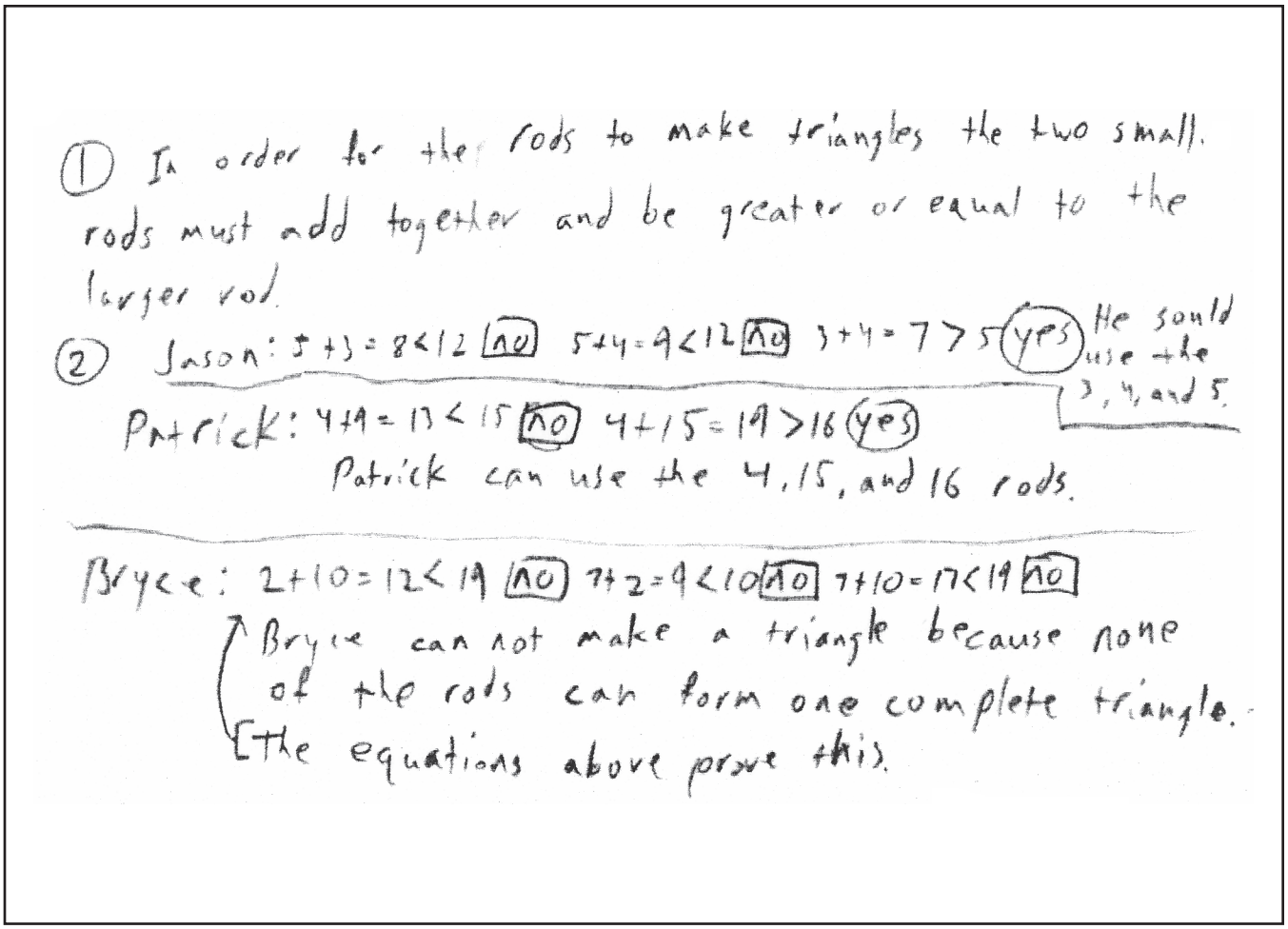
Part	Points
1	**1 point possible** 1 point: **Correct name, statement of, or explanation of theorem.** Give credit for the following or equivalent: • The Triangle Inequality Theorem, or • "The sum of the lengths of **any** two sides of a triangle is greater than the length of the third side" (give credit if "any" is omitted), or • "Take the two shortest lengths and add them together. That number must be bigger than the third length," or • "The length of a side of a triangle must be between the sum and difference of the other two lengths." Note: No credit for the Pythagorean Theorem or its statement.
2	**3 points possible** Note: No credit will be awarded in Part 2 for answer or procedure if support is completely invalid (obviously incorrect procedure) in Parts 1 and/or 2 with no correct procedure shown. Ex: "Jason could use 3-, 4-, and 5-inch rods because they're closest together." (True, but insufficient.) Ex: "Patrick could use 4-, 15-, and 16-inch rods because you could make a right triangle." (Not true.) ½ point: **Correct combination for Jason: 3-, 4-, and 5-inch rods.** AND ½ point: **Correct and complete procedure shown.** Work may contain a calculation or copy error if answer is incorrect. Give credit for the following or equivalent: • $5 + 3 > 4$, $5 + 4 > 3$, $3 + 4 > 5$, or • $3 + 4 > 5$, or • $5 + 4 = 9 > 3$ and $5 - 4 = 1 < 3$. (If using this method, it is not necessary to check all three pairs.) Note: If student uses the Pythagorean Theorem in Part 1, then credit will be awarded for 3, 4, 5 with or without correct procedure shown. AND ½ point: **1 correct combination for Patrick: 4-, 15-, and 16-inch rods or 9-, 15-, and 16-inch rods.** AND ½ point: **Correct and complete procedure shown for appropriate combination.** Work may contain a calculation or copy error if answer is incorrect. Give credit for the following or equivalent: • $4 + 15 > 16$, $4 + 16 > 15$, $15 + 16 > 4$, or • $9 + 15 > 16$, $9 + 16 > 15$, $15 + 16 > 9$, or • $4 + 15 > 16$ or $9 + 15 > 16$, or • $4 + 16 = 20 > 15$ and $16 - 4 = 12 < 15$, or • $16 + 15 = 31 > 9$ and $16 - 15 = 1 < 9$. AND ½ point: **Correct response for Bryce: No combination works.** AND ½ point: **Give credit for the following or equivalent:** • All 4 combinations are checked: 7, 2, 10 : $7 + 2 < 10$ 7, 2, 19 : $7 + 2 < 19$ 2, 10, 19 : $2 + 10 < 19$ 7, 10, 19 : $7 + 10 < 19$, or • The smallest three numbers and the largest three numbers are checked as follows: 7, 2, 10: $7 + 2 < 10$ (Work for 7, 2 ,19 is not required.) 7, 10, 19: $7 + 10 < 19$ (Work for 2, 10, 19 is not required.)



SCORE: 4

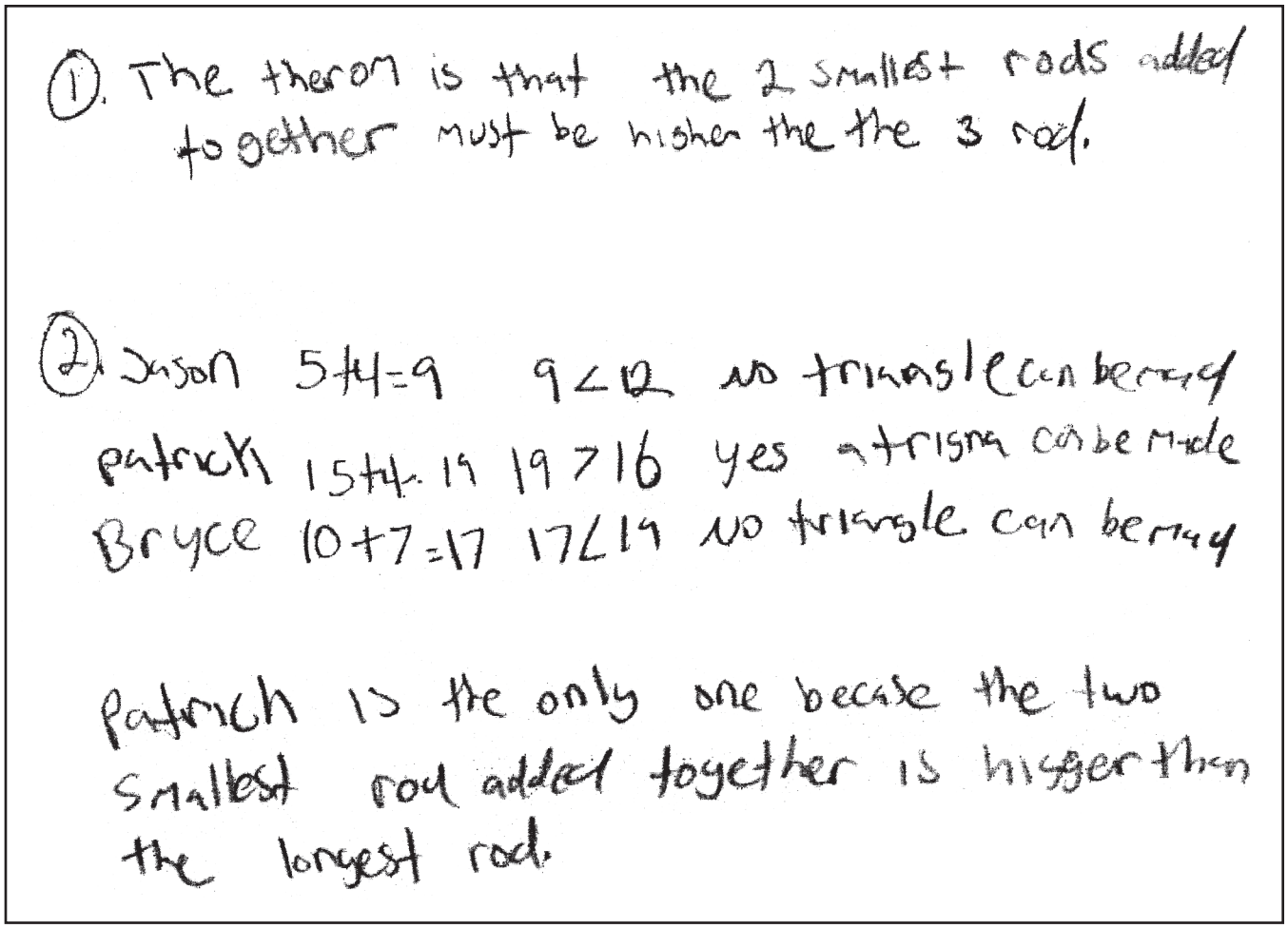
Part 1:		**Points**
Correct answer:	"The sum of the two smaller rods must add up to be more than the larger rod."	1

Part 2:		
Correct answer (Jason):	3, 4, 5	½
Correct and complete procedure (based on Part 1):	3 + 4 = 7 > 5	½
Correct answer (Patrick):	9, 15, 16	½
Correct and complete procedure:	9 + 15 = 24 > 16	½
Correct answer (Bryce):	None	½
Correct and complete procedure:	7, 10, 19 : 7 + 10 = 17 < 19 2, 10, 19 : 2 + 10 = 12 < 19 2, 7, 10 : 2 + 7 = 9 < 10 2, 7, 19 : 2 + 7 = 9 < 19	½
TOTAL POINTS:		**4**



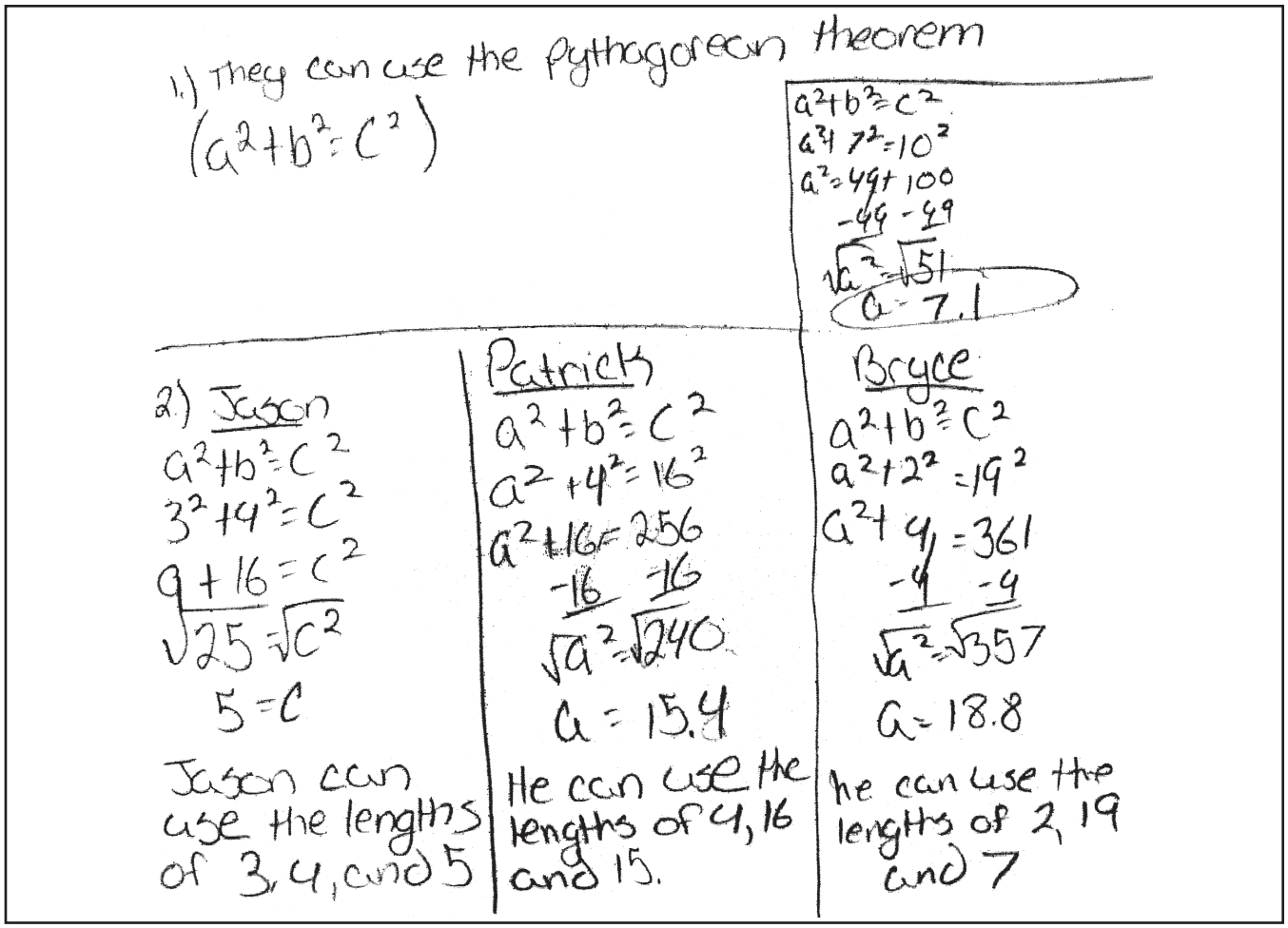
SCORE: 3

Part 1:		**Points**
Incorrect answer:	"be greater or equal to the larger rod"	-
Part 2:		
Correct answer (Jason):	3, 4, and 5	½
Correct and complete procedure:	3 + 4 = 7 > 5	½
Correct answer (Patrick):	4, 15, 16	½
Correct and complete procedure:	4 + 15 = 19 > 16	½
Correct answer (Bryce):	"can not make a triangle"	½
Correct and complete procedure:	2 + 10 = 12 < 19, 7 + 2 = 9 < 10 7 + 10 = 17 < 19 Not stated: If 7 = 2 < 10, then 7 + 2 must be less than 19.	½
TOTAL POINTS:		**3**



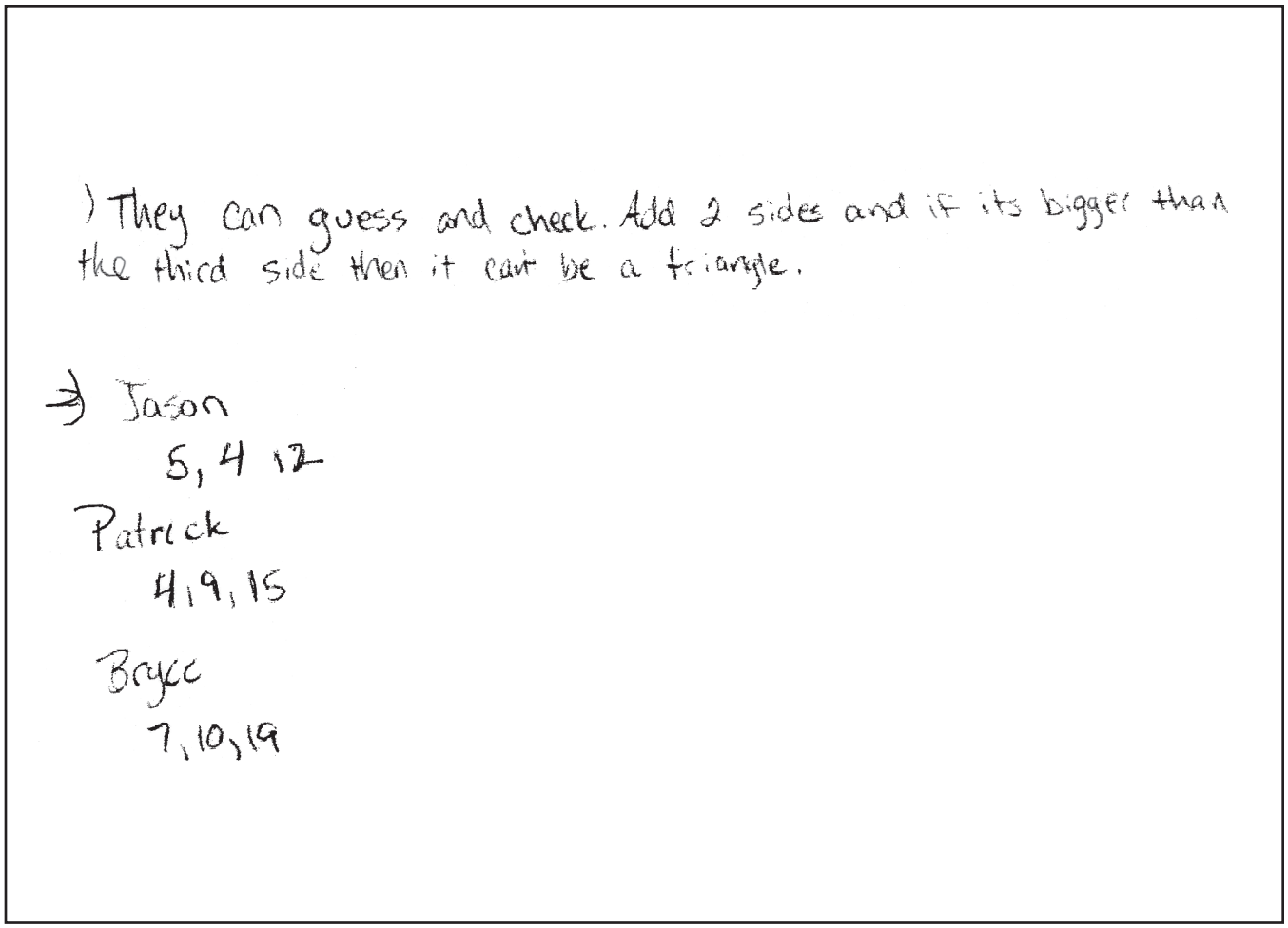
SCORE: 2

Part 1:		**Points**
Correct answer:	"the 2 smallest rods added together must be higher the the 3 rod."	1
Part 2:		
Incorrect answer (Jason):	"No triangle can be mad"	-
Incorrect procedure:	5 + 4 = 9, 9 < 12 (Compares to largest value given.)	-
Incorrect answer (Patrick):	"a triangle can be made" (No combination stated.)	-
Correct and complete procedure:	15 + 4 = 19, 19 > 16	½
Correct answer (Bryce):	"No triangle can be mad"	½
Incomplete procedure:	10 + 7 = 17, 17 < 19 (Examines only 7, 10, 19.)	-
TOTAL POINTS:		**2**



SCORE: 1

Part 1:		**Points**
Incorrect answer:	Pythagorean Theorem	-
Part 2:		
Correct answer (Jason):	3, 4, 5	½
Correct and complete procedure (based on Part 1):	Uses converse of Pythagorean Theorem $3^2 + 4^2 = c^2$, $5 = c$	½
Neither answers nor procedures for Patrick and Bryce are a factor in scoring since Part 1 is completely incorrect.		-
TOTAL POINTS:		**1**



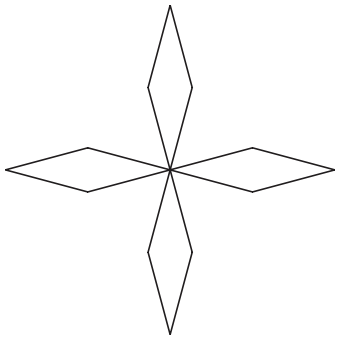
SCORE: 0

Part 1:		**Points**
Incorrect answer:	"They can guess and check. Add 2 sides and if its bigger than the third side then it can't be a triangle."	-

Part 2:		
Incorrect answer (Jason):	5, 4, 12	-
Missing procedure:		-
Incorrect answer (Patrick):	4, 9, 15	-
Missing procedure:		-
Incorrect answer (Bryce):	7, 10, 19	-
Missing procedure:		-
TOTAL POINTS:		**0**

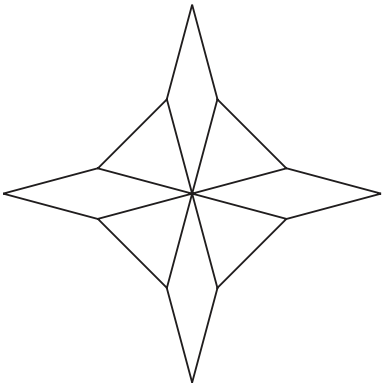
Item E—2007 Geometry

Jessica is making a window ornament from pieces of stained glass in the shapes of rhombi and triangles. Jessica begins the ornament with four rhombus-shaped pieces arranged at equal rotations around a center point, as shown below.



1. The smallest angles of each rhombus piece measure 30 degrees. In your answer document, draw one of the rhombi pieces and label all of its angle measures. Show all of your work and/or explain your answer.

Next, Jessica places triangular pieces between the rhombi pieces, connecting the vertices formed by the largest angles of each rhombus, as shown below.

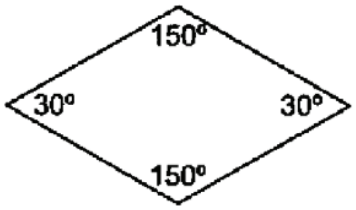


2. In your answer document, draw one of the triangular pieces. Label all of its angle measures. Show all of your work and/or explain your answer.

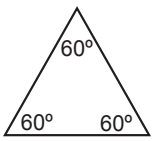
Item E Scoring Rubric—2007 Geometry

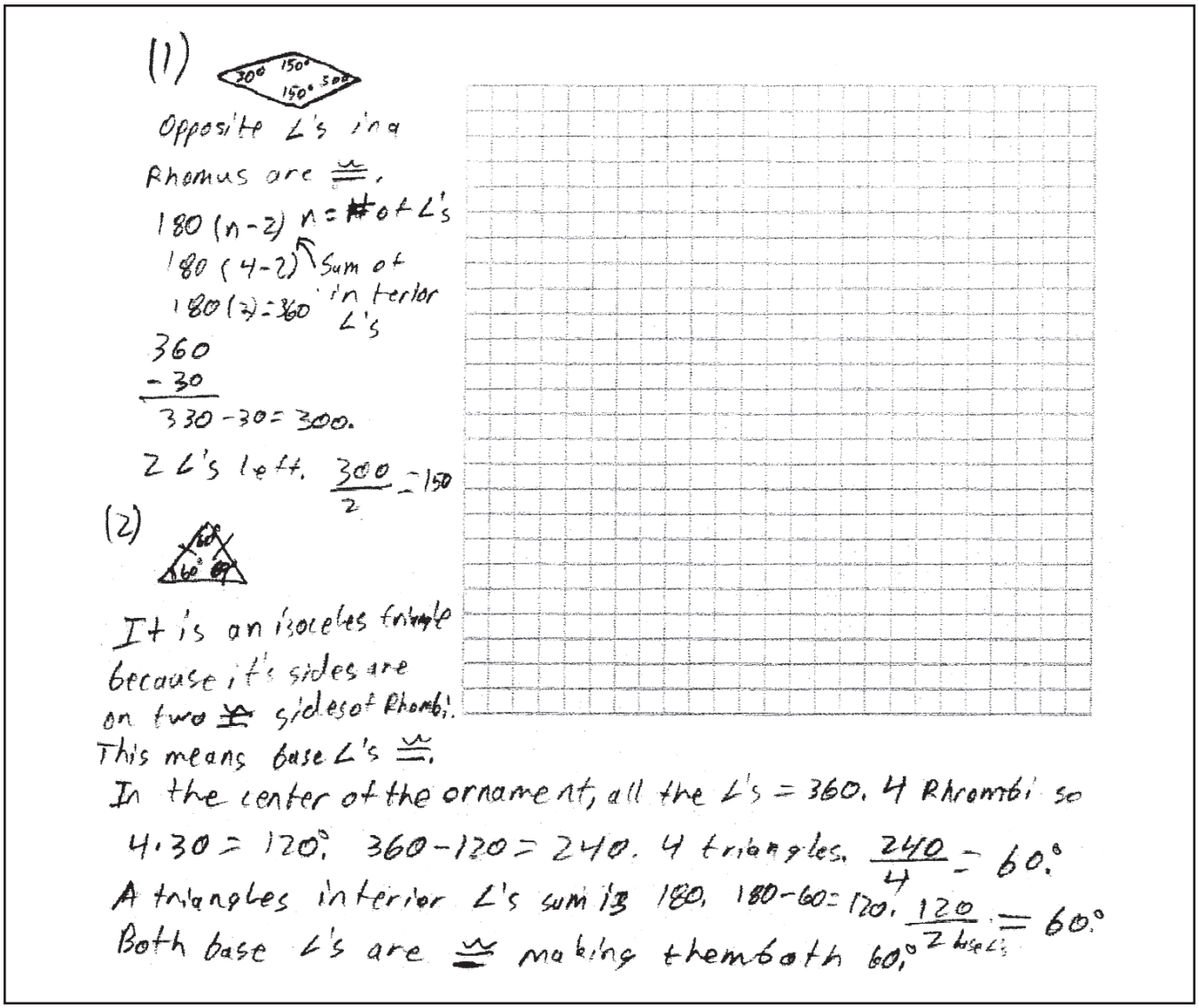
SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work. The label of "degrees" or " ° " is included somewhere in the response. An explanation of why the measures of the center angles of one triangle + one rhombus = 90° is included in Part 2.
3	The student earns 4 points.
2	The student earns 2–3 points.
1	The student earns 1 point, or some minimal understanding is shown.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.).

Solution and Scoring

Part	Points
1	**2 points possible** 1 point: **Drawing of a rhombus with correct and complete angle measures as shown below.** Note: The "degree" label is required only for a score of 4.  AND 1 point: **Correct and complete procedure shown and/or explained.** Work may contain a calculation or copy error. Give credit for the following or equivalent: • (All angles in a quadrilateral must add to 360º) 30 + 30 + x + x = 360, 60 + 2x = 360 2x = 300, x = #, or • (n – 2)180 = (4 – 2)180 = 360 360 – 60 = 300, 300/2 = #, or • "In a rhombus, consecutive angles are supplementary, so I know the other angle is 150 degrees."

Solution and Scoring (continued)

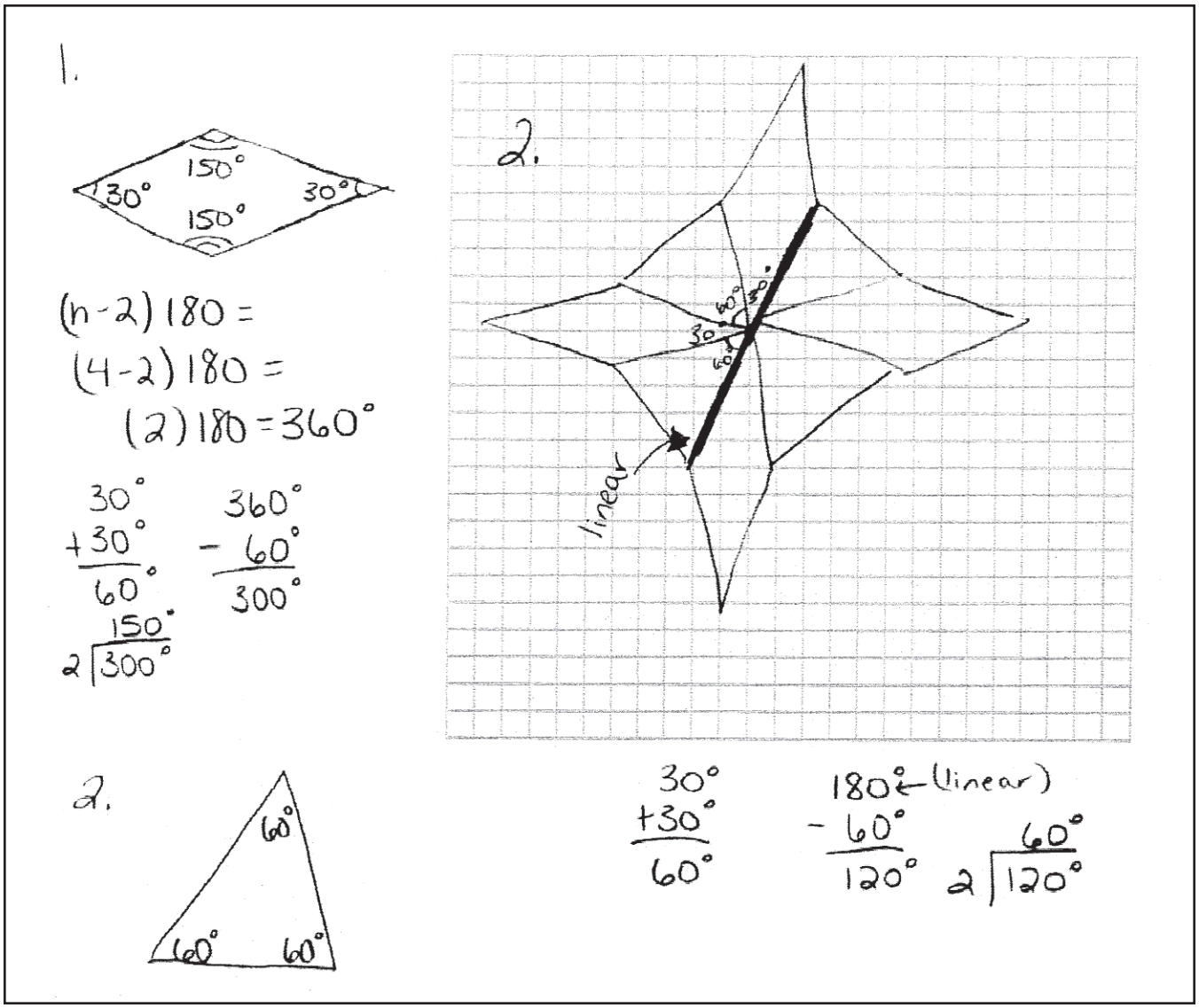
Part	Points
2	**3 points possible** 1 point: Drawing of an equilateral triangle with correct and complete angle measures as shown below. Note: The "degree" label is required only for a score of 4.  AND 1 point: **Correct and complete procedure shown and/or explained finding the measure of the angle of the triangle whose vertex is the center point.** Work may contain a calculation or copy error. Give credit for the following or equivalent: • (All angles about the center point must add to 360º) x = measure of angle of triangle whose vertex is the center point 30 + 30 + 30 + 30 + x + x + x + x = 360 120 + 4x = 360 4x = 240 x = 60, or • 2(30) + 2x = 180 2x = 120 x = 60, or • "The angles of the rhombus and the triangle whose vertex is the center point make a 90º angle." (Explanation of why this is true is required for a score of 4.) "So the measure of the angle of the triangle is 90 – 30 = 60º." AND 1 point: **Statement or drawing showing that the <u>triangle is isosceles</u> and the base angles measure 60º (work may be missing).** Work may contain a calculation or copy error or may be based on an incorrect measure of the "center angle" (work must be shown to receive credit). Give credit for the following or equivalent: • "<u>Because the sides shared with the rhombi are congruent, the triangles are isosceles</u>. The base angles must be congruent and the sum of the angles in the triangle must be 180°. I subtract 60 from 180 and get 120. Half of 120 is 60 for each base angle," or • Diagram shows that 2 sides are of the triangle $\cong$ and the base angles measure 60º (with "center angle" measuring 60º), or • "The <u>triangle is isosceles</u>. Since the 1st angle is 60º, I know the base angles are each 60º since 60 + 60 + 60 = 180, the triangle is equilateral." **Note: Do not give credit if the response jumps to the conclusion that the triangle is equilateral (or marks 3 sides $\cong$ on a diagram) without initially stating or showing that 2 sides are $\cong$.** Ex: "Measure of base angles = 60º since 180/3 = 60" Ex: "It's an equilateral triangle so all angles are 60 degrees." (Does not initially state that the triangle is isosceles.)



SCORE: 4

Part 1:		**Points**
Correct and complete drawing:		1
Correct and complete procedure:	180(4 – 2) = 360, 360 – 30 = 330 330 – 30 = 300, 300 ÷ 2 = 150	1

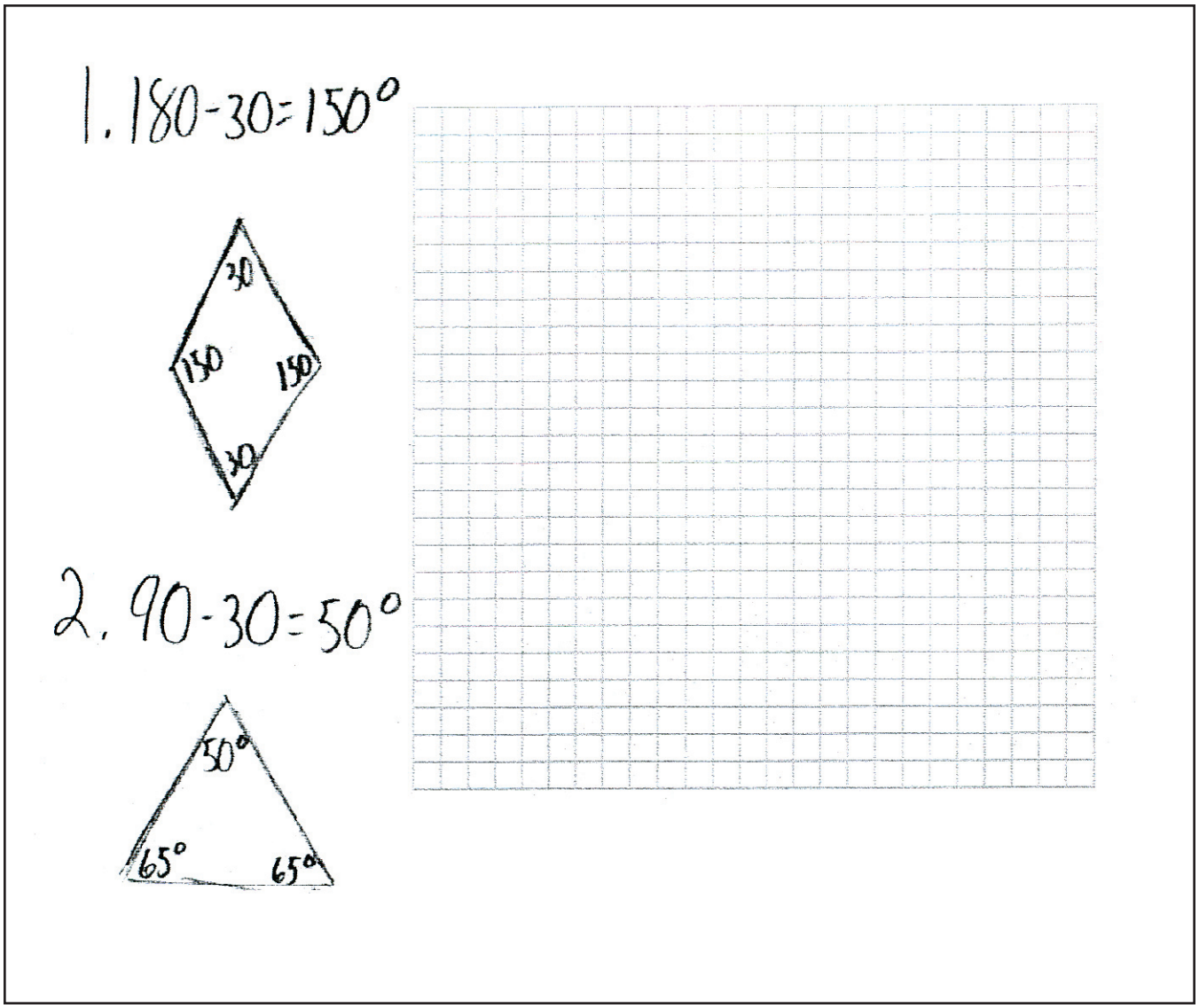
Part 2:		
Correct and complete drawing:		1
Correct and complete procedure "center" angle:	4 × 30 = 120, 360 – 120 = 240, 240 ÷ 4 = 60	1
Correct and complete procedure base angles:	Sides ≅ so base ∠s ≅, Sum ∠s = 180, 180 – 60 = 120, 120 ÷ 2 = 60	1
TOTAL POINTS:		**5**



SCORE: 3

Part 1:		**Points**
Correct and complete drawing:		1
Correct and complete procedure:	(4 – 2)180 = 360, 30 + 30 = 60 360 – 60 = 300, 300 ÷ 2 = 150	1

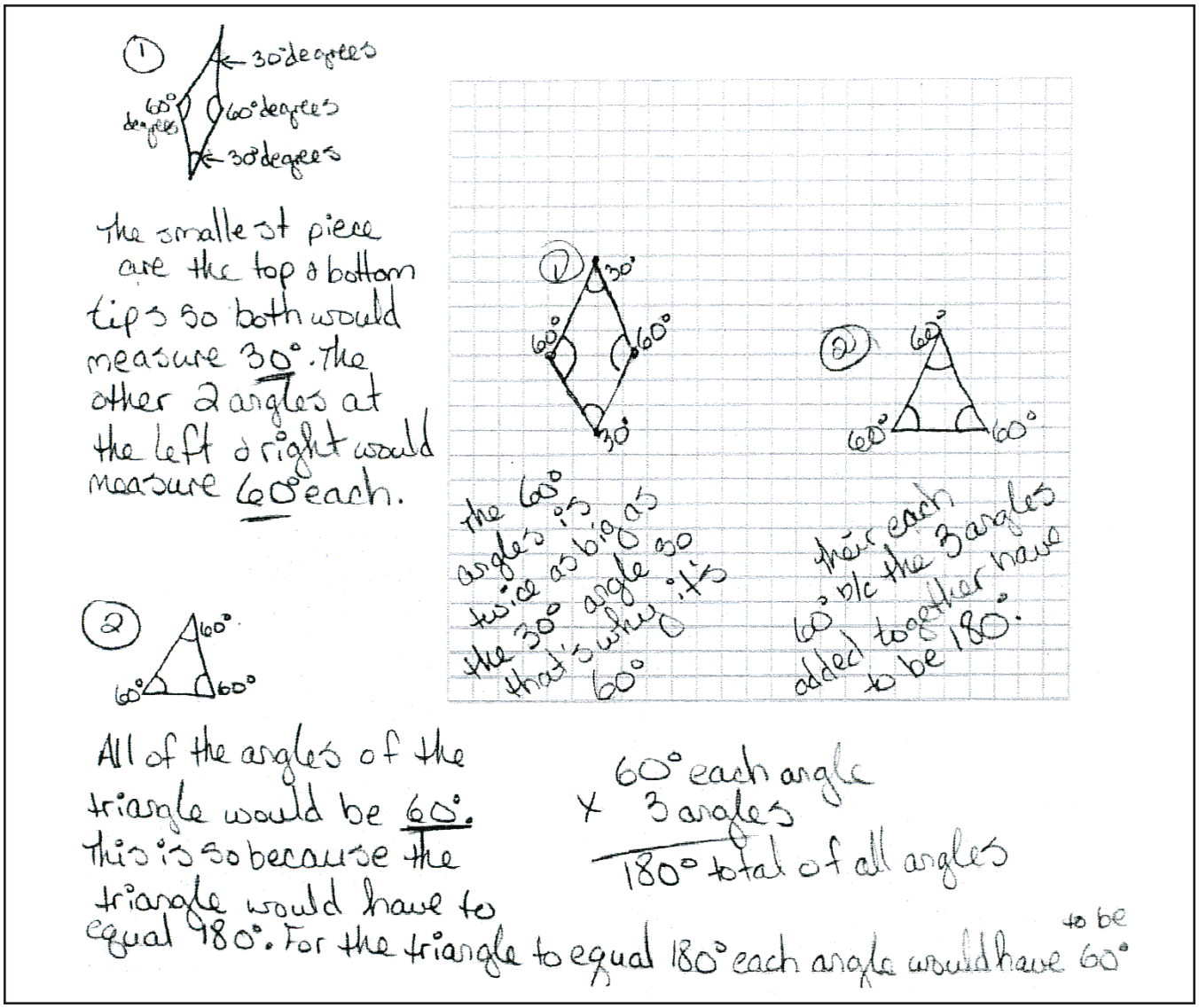
Part 2:		
Correct and complete drawing:		1
Correct and complete procedure "center" angle:	30 + 30 = 60, 180 – 60 = 120, 120 ÷ 2 = 60º	1
Missing procedure base angles:		-
TOTAL POINTS:		**4**



SCORE: 2

Part 1:		**Points**
Correct and complete drawing:		1
Correct and complete procedure:	180 – 30 = 150º	1

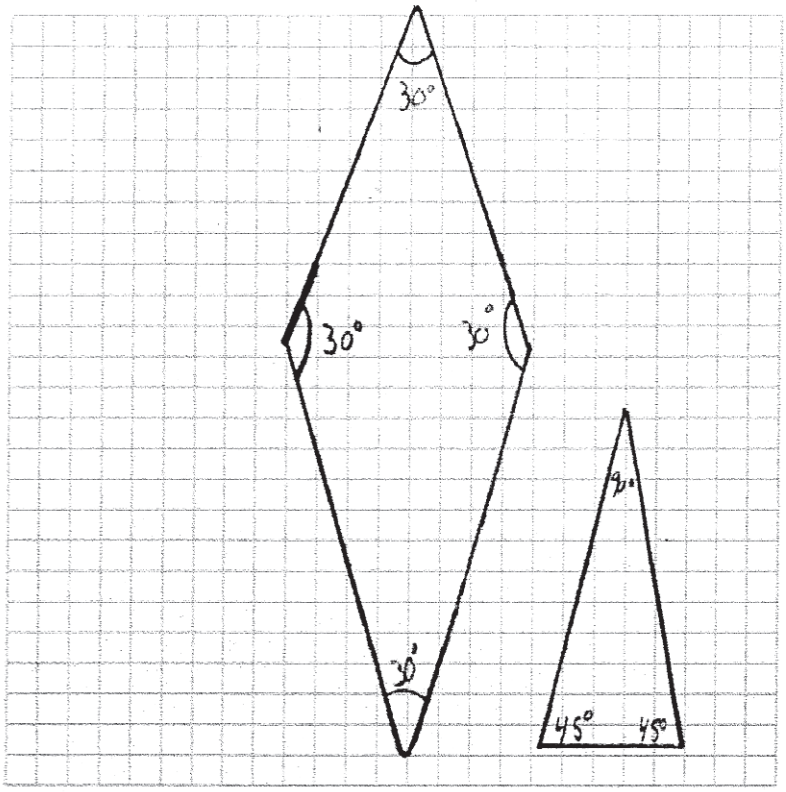
Part 2:		
Incorrect drawing (due to calculation error):		-
Correct and complete procedure "center" angle:	90 – 30 = 50º (calculation error)	1
Incomplete procedure base angles:	65º (No work finding measure or why ∠s ≅)	-
TOTAL POINTS:		**3**



SCORE: 1

Part 1:		**Points**
Incorrect drawing:		-
Incorrect procedure:	"The 60º angles is twice as big as the 30º angle..."	-
Part 2:		
Correct and complete drawing:		1
Missing procedure "center" angle:	All ∠s would be 60, because the Δ = 180	-
Incomplete procedure base angles:	60 × 3 = 180 (why ∠s ≅ ?)	-
TOTAL POINTS:		**1**

1.) The passage said that each piece was 30°s.



2) This is a 45°-45°-90° triangle

SCORE: 0

Part 1:		**Points**
Incorrect drawing:		-
Incorrect procedure:	"each piece was 30ºs."	-
Part 2:		
Incorrect drawing:		-
Missing procedure "center" angle:	90º—no work shown	-
Missing procedure base angles:	45º—no work shown	-
TOTAL POINTS:		**0**

ACTAAP

Arkansas Comprehensive Testing, Assessment, and Accountability Program

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